<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>/25</td>
</tr>
<tr>
<td>2.</td>
<td>/20</td>
</tr>
<tr>
<td>3.</td>
<td>/20</td>
</tr>
<tr>
<td>4.</td>
<td>/20</td>
</tr>
<tr>
<td>5.</td>
<td>/15</td>
</tr>
</tbody>
</table>

Instructions:

i. Closed book, closed notes. One 8.5” by 11” study sheet allowed. All work on the exam must be your own.

ii. Write your solution work on one side of the page only. Do not write on the back of any pages.

iii. Please be neat. Only neat answers will be granted partial credit.

iv. Significant figures count.

v. Please box your final answers.

vi. If you use a calculator, please write your steps if you wish partial credit.

vii. No cell phones or internet-capable instruments allowed.

There are five problems on the test.

There are three pages to the test.

There is an additional handout of resources (10 pages).
This page is intentionally blank.
1. (25 points) A very tall, very wide slab (thermal conductivity \( k \)) of thickness \( B \) is positioned between two fluids as shown in the figures below. For each of the following five situations, sketch the steady state temperature profile, both in the fluids and in the slab itself. Use the axes shown and draw your answers carefully.

   a. Boundary conditions are specified that the left wall temperature (and left fluid temperature) is \( T_1 \) and the right wall temperature (and right fluid temperature) is \( T_2 \).

   b. Same as a), except \( k \) of the slab is twice as large.

   c. Boundary conditions are specified that the bulk fluid temperature on the left is \( T_1 \) and the bulk fluid temperature on the right is \( T_2 \). Heat transfer coefficients \( h_1 \) (left side) and \( h_2 \) (right side) are finite.

   d. Same as c), except \( h_1 \) is very large (infinite).

   e. Same as c), except \( k \) is 1/10th as large as when you drew your c) answer. In the box below, describe in words what is different about the curve you drew for part e) compared to part c).

---

Compare e) sketch with c):
2. (20 points) How much energy does it take to heat $5.0 \times 10^{-1}$ kg/s water (thermal conductivity = 0.608 $W/mK$, heat capacity = 4.182 $kJ/kg K$) from 15.0°C to 34°C? Please give your answer in SI units (metric system) and box your answer.

3. (20 points) A solid wall (thermal conductivity = 1.212 $W/mK$, heat capacity = 2.11 $kJ/kg K$, wall thickness = 8.2 cm) has a steady-state temperature gradient of $(-2.3 \times 10^3 \ K/m)$. What is the steady state heat flux through the wall? Please give your answer in SI units (metric system) and box your answer.

4. (20 points) For a 1.0 mm diameter polystyrene bead (density = 1002.06 $kg/m^3$, molecular weight = $30 \times 10^3 g/mol$) falling in Superfluid® (density = 997.08 $kg/m^3$; viscosity = $1.390 \times 10^{-3} \ Pa s$; molecular weight = 32.12 $g/mol$), what is the drag on the sphere if it falls at a Reynolds number of $Re = \rho V D/\mu = 1.4$? Please give your answer in Newtons and box your answer.

5. (15 points) A long metal pipe of inside radius $R_1$ and outside radius $R_2$ has its inner surface maintained at $T_1$ and its outer surface maintained at $T_2$. The thermal conductivity of the metal varies with temperature according to $k = a + bT + cT^3$ (where $a, b, c$ are all constants). What is the steady state radial heat flux $\bar{q}_r = q_r/A$ equal to in this case? Solve for your answer in terms of integration constants and boundary conditions (you do not need to do the remaining algebra).
Exam 4  
CM3110  
Fall 2014  
Solution

1. a) See next page  
   (uniform profile between \( T_1 \) and \( T_2 \))

b) Same - \( k \) does not affect \( \frac{dT}{dx} \) with these boundary conditions.

c) finite offset at both edges

d) no offset left edge, finite offset right edge

e) Sketch similar to c,
   slope may or may not change depending on how \( h_1, h_2 \) adjust.

SEE SKETCH
1. (25 points) A very tall, very wide slab (thermal conductivity = $k$) of thickness $B$ is positioned between two fluids as shown in the figures below. For each of the following five situations, sketch the steady state temperature profile, both in the fluids and in the slab itself. Use the axes shown and draw your answers carefully.

a. Boundary conditions are specified that the left wall temperature (and left fluid temperature) is $T_1$ and the right wall temperature (and right fluid temperature) is $T_2$.

b. Same as a), except $k$ of the slab is twice as large.

c. Boundary conditions are specified that the bulk fluid temperature on the left is $T_1$ and the bulk fluid temperature on the right is $T_2$. Heat transfer coefficients $h_1$ (left side) and $h_2$ (right side) are finite.

d. Same as c), except $h_1$ is very large (infinite).

e. Same as c), except $k$ is $1/10^{th}$ as large as when you drew your c) answer. In the box below, describe in words what is different about the curve you drew for part e) compared to part c).

---

Compare e) sketch with c):

Slope $\frac{dT}{dx}$ may be smaller, the same, or larger when $k$ is smaller. It depends on how $h_1, h_2$ adjust.
\[ Q_{\text{in}} = m(\hat{H}_{\text{out}} - \hat{H}_{\text{in}}) \]

\[ = m \Delta H \text{ (water, } c_p \text{ constant, } T \text{ changes)} \]

\[ = m c_p (T_{\text{out}} - T_{\text{in}}) \text{ (see handout)} \]
\[ Q = \left( \frac{0.5 \text{ kg}}{5} \right) \left( \frac{4.182 \text{ kJ}}{\text{kg K}} \right) \left( 34 - 15 \right) \text{ K} \]

\[ = \frac{39.7290 \text{ kJ}}{5} \]

\[ = 40 \text{ kW} = \frac{4.0 \times 10^4 \text{ kW}}{40,000 \text{ W}} \]

**Note:**

\[ ^\circ \text{C} \]

\[ 34 + 273.16 = 307.16 \text{ K} \]

\[ 15 + 273.16 = 288.16 \text{ K} \]

\[ 19 \text{ K} \]

(The Kelvin part is irrelevant when temp differences are considered.)
Steady state heat flux:

\[-k \frac{dT}{dx} = \frac{Q_x}{A}\]

\[\frac{Q_x}{A} = (-1) \left( 1.212 \frac{W}{mK} \right) \left( -2.3 \times 10^3 \frac{K}{m} \right)\]

\[= 2.7876 \times 10^3 \frac{W}{m^2}\]

\[= 2.8 \text{ kW/m}^2\]
Polystyrene

\[ D = 1 \text{ mm} = 10^{-3} \text{ m} \]

\[ \rho = 1002.06 \text{ kg/m}^2 \text{ body} \]

\[ \rho_{\text{fluid}} = 997.068 \text{ kg/m}^3 \]

\[ M_{\text{fluid}} = 1.390 \times 10^3 \text{ kg/m s} \]

\[ Re = \frac{\rho V D}{\mu} = 1.4 \implies \text{creeping flow} \]

Calc \[ \frac{V_{\infty}}{\alpha} = V \]

\[ Re = \frac{\rho V D}{\mu} \]

\[ 1.4 = \frac{(997.068 \text{ kg/m}^3)(V)(10^{-3})}{1.390 \times 10^3 \text{ kg/m s}} \]

\[ V = \frac{V_{\infty}}{\alpha} = 1.951698 \times 10^{-3} \text{ m/s} \]
An airming flow

\[ C_D = \frac{2\nu}{\text{Re}} = \frac{F_{\text{drag}}}{\frac{1}{2} \rho v_\infty^2 \pi R^2} \]

\[ F_{\text{drag}} = \frac{1}{2} \rho v_\infty^2 \pi R^2 \frac{2\nu}{\text{Re}} \]

\[ = \left( \frac{1}{2} \right) \left( \frac{997.08 \times 10^3}{\text{kg/m}^3} \right) \left( 1.951698 \times 10^{-3} \text{ m} \right)^2 \frac{2 \times 5}{1.4 \times 10^{-3}} \]

\[ = 2.5568 \times 10^{-5} N = \boxed{2.6 \times 10^{-5} N} \]

**OR**

\[ F_{\text{drag}} = 6\pi \nu R u_0 \quad (\text{Stokes Einstein}) \]

\[ = (6)(\pi) \left( \frac{10^{-3}}{2} \right) \left( \frac{1390 \times 10^{-3}}{\text{kg/m}^3} \right) \left( 1.951698 \times 10^{-3} \right) \]

\[ \times \frac{N \times 8}{\text{kg} \times \text{m}} \]

\[ = 2.5568 \times 10^{-5} N \]

\[ = 2.6 \times 10^{-5} N \]
\[ R = a + bT + cT^3 \]

\[ \frac{\partial}{\partial r} = \frac{\partial r}{A} = ? \]

**Micro Energy Eqn**

\[ \rho C_p \left( \frac{\partial T}{\partial t} + v \cdot \nabla T \right) = -D \cdot \nabla^2 T + J \]

- Steady state
- Solid pipe \( V = 0 \)
- No reaction
- No current
\[ \frac{\partial r}{A} = \frac{Q}{r} = -k(t) \frac{dT}{dr} \left( a + bT + cT^3 \right) \]

\[ \frac{c}{r} = -(a + bT + cT^3) \frac{dT}{dr} \]

Separate the variables \( T, r \):

\[ (-c) \frac{dr}{r} = (a + bT + cT^3) dT \]

Integrate both sides:

\[ (-c) \ln r = aT + bT^2 \frac{2}{2} + cT^4 \frac{4}{4} + C_2 \]

\[ \text{BC: } r = R_1 \quad T = T_1 \]
\[ r = R_2 \quad T = T_2 \]

Solve for \( C_1, C_2 \):

\[ \frac{Q}{A} = C_1/r \]
Solution (to the end):

\[-G_1 \ln R_1 = aT_1 + b \frac{T_1^2}{2} + c \frac{T_1}{y} + c_2\]

\[-G_1 \ln R_2 = aT_2 + b \frac{T_2^2}{2} + c \frac{T_2}{y} + c_2\]

Subtract
\[-G_1 (\ln R_2 - \ln R_1) = c_1 \ln \frac{R_2}{R_1}\]

\[-G_1 (\ln R_1 - \ln R_2)\]

\[= a(T_1 - T_2) + b \left( \frac{T_1^2}{2} - \frac{T_2^2}{2} \right)\]

\[+ \frac{c}{y} (T_1 - T_2)\]

\[G_1 = \frac{a(T_1 - T_2) + b \left( \frac{T_1^2}{2} - \frac{T_2^2}{2} \right) + \frac{c}{y} (T_1 - T_2)}{\ln \frac{R_2}{R_1}}\]

Substitute back into one of the above to get an expression for $c_2$ (bit messy; we don't need it)
What is \( \frac{Q_r}{A} \)?

\[
\frac{Q_r}{A} = \frac{c_1}{r}
\]

\[
\frac{Q_r}{A} = \frac{1}{r} \left( \frac{\alpha (T_1 - T_2) + \frac{6}{2} (T_1^2 - T_2^2) + \frac{\epsilon}{T} (T_1 - T_2)}{\ln \frac{R_2}{R_1}} \right)
\]