i. Closed book, closed notes. One 8.5” by 11” study sheet allowed. All work on the exam must be your own.

ii. Write your solution on one side of the page only. Do not write on the back of any pages.

iii. Please be neat. Only neat answers will be granted partial credit.

iv. Significant figures count.

v. Please box your final answers.

1. (25 points) Fluid (a) and fluid (b) are used in a manometer as shown below; the pressure on the top of the right side is $p_{atm} = 1.00\text{ atm}$. The density of fluid (a) is 1.000 g/cm$^3$ and the density of fluid b is 13.60 g/cm$^3$. What is the pressure $p$? Please give your answer in $\text{atm}$.

![Manometer Diagram]

Note: the inner diameter of the manometer tube is 11.2 mm.
3. (25 points) For the vectors $\mathbf{v}$ and $\mathbf{a}$ and matrix $\mathbf{M}$ given below, calculate the four indicated quantities. Show your work.

$$
\mathbf{v} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}_{xyz} \quad \mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}_{xyz} \quad \mathbf{M} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 3 \\ 2 & 0 & 5 \end{pmatrix}_{xyz}
$$

a. $\mathbf{v} \cdot \mathbf{a}$
b. $|\mathbf{v}|$ (the magnitude of $\mathbf{v}$)
c. $\mathbf{M} \cdot \mathbf{a}$
d. $\hat{\mathbf{a}}$ (a unit vector in the direction of $\mathbf{a}$)

4. (25 points) Water (25°C) flows steadily in a horizontal smooth pipe in turbulent flow. If the frictional losses are 32,000 $f t^2/s^2$, what is the pressure drop along the length of pipe in psi ($lb/ft^2$)?

5. (25 points) For the steady, downward, laminar flow of water through a long vertical pipe, the fluid velocity varies with position in the tube. With the methods of this course, we calculate the velocity as a function of position in cylindrical coordinates to be:

$$
\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ v_z \end{pmatrix}_{r\theta z} = v_z \hat{e}_z
$$

$$
v_z(r) = \frac{R^2(L\rho g + p_0 - p_L)}{4\mu L} \left(1 - \frac{r^2}{R^2}\right)
$$

Calculate the flow rate $Q$ from the velocity profile $v_z(r)$ above, using the integral given below:

$$
Q = \int_0^{2\pi} \int_0^R (v_z) \ rdrd\theta
$$

where $R$ is the pipe radius and $r, \theta, z$ are the usual cylindrical coordinates. Please show your work. The following quantities are constants:

$$
R = \text{pipe radius} \\
\rho = \text{fluid density} \\
g = \text{acceleration due to gravity} \\
p_L = \text{pressure at exit} \\
p_0 = \text{pressure at entrance} \\
\mu = \text{fluid viscosity} \\
L = \text{pipe length}
$$