Instructions:
   i. Closed book, closed notes. One 8.5” by 11” study sheet allowed. All work on the
      exam must be your own.
   ii. Write your solution work on one side of the page only. Do not write on the back
      of any pages.
   iii. Please be neat. Only neat answers will be granted partial credit.
   iv. Significant figures count.
   v. Please box your final answers.
   vi. If you use a calculator, please write your steps if you wish partial credit.
   vii. No cell phones or internet-capable instruments allowed.

There are five problems on the test.
There are three pages to the test.
There are two handouts of resources: One is the Navier-Stokes
   in three coordinates; the other is a 5 page formula sheet,
   double sided.

1. (20 points) What is the pressure drop in 200.0 meters of smooth horizontal
   copper tubing of inner diameter 1.5 cm = 0.015 m? Water at 25°C is flowing at
   1.31 × 10⁻² m/s average velocity. Please give your answer in Pa.

2. (20 points) The wetted surface area of a circular pipe is the inside surface; this
   area can be calculated by integrating using the equation below. For a pipe of
   length L and radius R, show how you can arrive at the correct surface area by
   integrating this equation using cylindrical coordinates.

   \[ \text{surface area} = \int_{Area} dA \]
3. (20 points) An incompressible, Newtonian fluid (viscosity $\mu$, density $\rho$) flows steadily between two long, wide parallel plates under an imposed pressure difference, $\Delta p = P_0 - P_L$ (see below). In addition, the top plate moves at a velocity $V$ parallel to the bottom plate. What are the velocity and pressure boundary conditions for this flow? Express your answers mathematically in the coordinate system given. Be specific and complete.

4. (20 points) For pressure driven flow downward in a vertical tube, we solved in the homework for the velocity field as follows:

$$v(r) = \begin{pmatrix} 0 \\ 0 \\ \frac{P_0 - P_L + \rho g L R^2}{4 \mu L} \left(1 - \frac{r^2}{R^2}\right) r \theta z \end{pmatrix}$$

where $P_0$ is the upstream pressure, $P_L$ is the pressure a distance $L$ downstream, $R$ is the tube radius, $\mu$ is the viscosity, and $g$ is the acceleration due to gravity. The pressure field is given by

$$p = \frac{(P_L - P_0)}{L} z + P_0$$

a) What is the stress tensor $\Pi$ for this flow?

b) What is $\hat{n} \cdot \Pi$ evaluated at the inner surface of the wall?
5. (20 points) Calculate the velocity profile for pressure-driven flow of an incompressible Newtonian liquid in the annular gap between two long vertical cylinders, as shown below. The radius of the inner cylinder is $\kappa R$ ($\kappa < 1$) and the radius of the outer cylinder is $R$. The fluid is in the gap between the smaller inner cylinder and the larger outer cylinder. The pressure at an upstream point is $P_0$; at a point a distance $L$ downstream, the pressure is $P_L$. Assume that the flow is well developed and at steady state. You may neglect gravity. You may leave your answer in terms of the integration constants; you must provide the equations to be solved to obtain the integration constants.