

Energy Balance Notes CM2110/CM3110/CM3120

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Choosing the Right Energy Balance

1. Closed System (note: $\Delta = \Sigma_{final} - \Sigma_{initial}$)
 - $\Delta E_k + \Delta E_p + \Delta U = Q_{in} + W_{on}$ (FR)
 - Is it adiabatic? (if yes, $Q_{in}=0$)
 - Are there moving parts, e.g. do the walls move? (if no, $W_{on}=0$)
 - Is the *system* moving? (if no, $\Delta E_k=0$)
 - Is there a change in elevation of the *system*? (if no, $\Delta E_p = 0$)
 - Does T, phase, or chemical composition change? (if no to all, $\Delta U = 0$)

2. Open System (the fluid is the system) (note: $\Delta = \Sigma_{out} - \Sigma_{in}$)

1. Is it a Mechanical Energy Balance (MEB) problem? (turbulent, $\alpha=1$; laminar, $\alpha=0.5$; F = total frictional loss between inlet and outlet, $W_{s,on} = -W_{s,by}$)

- $\frac{\Delta P}{\rho} + \frac{1}{2\alpha}\Delta v^2 + g \Delta z + F = \frac{W_{s,on} fluid}{\dot{m}}$ (FR)
- $\frac{(P_{out}-P_{in})}{\rho} + \frac{(v_{out}^2-v_{in}^2)}{2\alpha} + g(z_{out} - z_{in}) + F = \frac{W_{s,on} fluid}{\dot{m}}$ (FR)

The mechanical energy balance is only valid for systems for which the following is true:

- i. single-input, single output
 - ii. small or zero Q_{in}
 - iii. incompressible fluid ($\rho = \text{constant}$)
 - iv. small or zero ΔT
 - v. no reaction, no phase change
2. Is it a *regular* open system balance?
 - $\Delta E_k + \Delta E_p + \Delta H = Q_{in} + W_{s,on}$ (FR)
 - Is it adiabatic? (if yes, $Q_{in} = 0$)
 - Are there moving parts, e.g. pump, turbine, mixing shaft? (if no, $W_{s,on} = 0$)
 - Does the average velocity of the fluid change between the input and the output? (if no, $\Delta E_k = 0$); remember $\langle v \rangle = v_{av} = v = \frac{\text{volumetric flow rate}}{\text{area}}$
 - Is there a change in elevation of the system between input and output? (if no, $\Delta E_p = 0$)
 - Does T , phase, chemical composition, or **P change**? (if no to all, $\Delta H = 0$)

Calculating Internal Energy

1. Constant T, **P changes only**
 - (a) real gases => look it up in a table (e.g. **steam**, Tables B5, B6, B7)
 - (b) ideal gases => $\Delta \hat{U} = 0$
 - (c) liquids, solids => $\Delta \hat{U} = 0$
2. Constant P, **T changes only**
 - (a) real gases => look it up in a table (e.g. **steam**), or, if V is constant, $\Delta \hat{U} = \int_{T_1}^{T_2} \hat{C}_v(T) dT$
 - (b) ideal gases => $\Delta \hat{U} = \int_{T_1}^{T_2} \hat{C}_v(T) dT$; also, $\hat{C}_p = \hat{C}_v + R$
 - (c) liquids, solids => $\Delta \hat{U} = \int_{T_1}^{T_2} \hat{C}_v(T) dT$; also $\hat{C}_p \approx \hat{C}_v$
3. Constant T, P, **phase changes**
 - (a) real gases => look it up in a table (e.g. **steam**)
 - (b) liquid to vapor => $\Delta \hat{U} = \Delta \hat{H}_{vap}(T) - P\Delta \hat{V}_{vap} \approx \Delta \hat{H}_{vap} - RT$
 - (c) solid to vapor => $\Delta \hat{U} = \Delta \hat{H}_{sub}(T) - P\Delta \hat{V}_{sub} \approx \Delta \hat{H}_{sub} - RT$
 - (d) solid to liquid => $\Delta \hat{U} = \Delta \hat{H}_{melt}(T) - P\Delta \hat{V}_{melt} \approx \Delta \hat{H}_{melt}$
4. Constant T, P, **mixing occurs**
 - (a) gases => $\Delta \hat{U} = 0$

(b) similar liquids $\Rightarrow \Delta \hat{U} = 0$

(c) dissimilar liquids/solids $\Rightarrow \Delta \hat{U} = \Delta \hat{H}_{solution}$, see WT or Perry's Handbook

Note: be careful with units, $\Delta \hat{H}_{solution} [=] \frac{kJ}{mole\ solute}$

5. Constant T, P, **reaction occurs**: $\Delta \hat{U} = \Delta \hat{H}_{rxn}$

Calculating Enthalpy

1. Constant T, **P changes only** (Note: Since T is constant, \hat{U} does not change.)

(a) real gases - look it up in a table (e.g. **steam**, Tables B5, B6, B7)

(b) ideal gases

$$\begin{aligned}\hat{H} &= \hat{U} + P\hat{V} \\ &= \hat{U} + RT \\ (\hat{H}_2 - \hat{H}_1) &= (\hat{U}_2 - \hat{U}_1) + R(T_2 - T_1) \\ \Delta \hat{H} &= \Delta \hat{U} = 0\end{aligned}$$

(c) liquids, solids

$$\begin{aligned}\hat{H} &= \hat{U} + P\hat{V} \\ \Delta \hat{H} &= \Delta(P\hat{V}) \\ \hat{V} &\approx \text{constant with respect to } P \\ \Delta \hat{H} &= \hat{V}(\Delta P)\end{aligned}$$

2. Constant P, **T changes only**

(a) real gases \Rightarrow look it up in a table to be most accurate (e.g. **steam**), otherwise $\Delta \hat{H} = \int_{T_1}^{T_2} \hat{C}_p(T) dT$

(b) ideal gases $\Rightarrow \Delta \hat{H} = \int_{T_1}^{T_2} \hat{C}_p(T) dT$

(c) liquids, solids $\Rightarrow \Delta \hat{H} = \int_{T_1}^{T_2} \hat{C}_p(T) dT$

3. Constant T, P, **phase changes**

(a) liquid to vapor $\Rightarrow \Delta \hat{H} = \Delta \hat{H}_{vap}(T)$ Note: $\frac{d \ln P^*}{d \ln(1/T)} = \frac{\Delta \hat{H}_{vap}}{R}$ (Clapeyron equation)

(b) solid to vapor $\Rightarrow \Delta \hat{H} = \Delta \hat{H}_{sub}(T)$

(c) solid to liquid $\Rightarrow \Delta \hat{H} = \Delta \hat{H}_{melt}(T)$

4. Constant T, P, **mixing occurs**

(a) gases $\Rightarrow \Delta \hat{H} = 0$

(b) similar liquids $\Rightarrow \Delta \hat{H} = 0$

(c) dissimilar liquids/solids $\Rightarrow \Delta \hat{H} = \Delta \hat{H}_{solution}$, see WT or Perry's; Note: be careful with units,

$$\Delta \hat{H}_{solution} [=] \frac{J}{mole\ solute}$$

5. Constant T, P, **reaction occurs**: $\Delta \hat{U} = \Delta \hat{H}_{rxn}$

Problem-Solving Strategies for Energy Balances

1. Write down *neatly* everything you are doing so that you and the grader both understand better what you are thinking.
2. Draw your flow sheet Large. Leave yourself plenty of room to add information to the drawing. Draw it over if it becomes too crowded. Do not erase; lightly cross out and start a new sheet. You may want to come back to the original information.
3. Always write complete units with quantities, e.g., for mole fraction A the units are $\frac{moles\ A}{moles\ total}$; quantities are meaningless without the units.

4. What are you looking for? What balance can help you find it?
 - a. Is a mass balance necessary?
 - b. Is it an open or a closed system?
 - c. Is it a mechanical energy balance problem?
5. Convert inconvenient units, e.g., convert volumes and volume fractions into moles or masses, since mass is conserved and volume is not. Also, convert dew points and other similar information (e.g. percent humidity, molal saturation, etc.) to compositions if possible.
6. Do you have a piece of information you do not know what to do with? What is its definition? Look it up in the index, if necessary. Write it with its proper units and try to interpret how it impacts the problem.
7. What has remained constant in the problem? Is it isothermal (T constant)? Isobaric (P constant)? Constant V or \hat{V} ? Adiabatic ($Q_{in} = 0$)? Is the mass flow constant? Is the volumetric flow constant? Is the heat flow constant or known?
8. Remember that if a system is **saturated**, you know a great deal about it:
 - a. If it is a *pure component*, you only need to know the phase (i.e. solid, liquid, vapor) and *one* of the following to know everything about the stream: $T, P, \hat{V}, \hat{U}, \hat{H}$.
 - b. If it is a *mixture*, Raoult's law applies to each component, $y_i P = x_i P_i^*(T)$.
9. When looking for \hat{U}, \hat{H} , or \hat{V} , always take it from a table, if it is available. It *is* available in a table for water/steam:
 - a. Table B.5, FR page 642, "Properties of *Saturated* Steam," sorted by temperature
 - b. Table B.6, FR page 644, "Properties of *Saturated* Steam," sorted by pressure
 - c. Table B.7, FR page 650, "Properties of *Superheated* Steam," presented in a grid of pressure and temperature. Saturated steam properties are also presented in the first two columns of Table B.7, but the steps in pressure are large, and therefore Tables B.5 and B.6 are more accurate for saturated steam properties at lower T and P .
10. If the problem is complex, break it down into smaller pieces and draw separate flow sheets that correspond to the smaller pieces.
11. Name unknown streams, compositions, and enthalpies or internal energies. See if there are few unknowns which can be solved for. Try different methods of naming the unknowns if the first way you think of does not turn out to be convenient.
12. Check for forgotten relations:
 - a. mass balance
 - b. mole fractions and mass fractions sum to 1.
 - c. If a stream is just split, with no special process unit present, the mole or mass fractions are the same in all streams before and after the split.
 - d. For a fixed, closed system, V, \hat{V} , and mass are constant.
13. The last step is to answer the question. Always present your answer with the correct number of significant figures and a box around it.

References

1. (FR) R. M. Felder, and R. W. Rousseau, *Elementary Principles of Chemical Processes*, 3rd Edition (Wiley, NY: 2000).
2. (G) C. J. Geankoplis, *Transport Processes and Unit Operations*, 4th Edition (Prentice Hall: Englewood Cliffs, NJ, 2003).
3. (WT) J. C. Whitwell and R. K. Toner, *Conservation of Mass and Energy*, pp 344-346, McGraw-Hill, Inc., 1969.