10.7 Summary

The modeling discussed in this text is the shared background of all flow modeling. From this starting point, we now can pursue more complex flow issues, from plastics processing to aeronautics. In this text we also focused on revealing a process of problem solving, in which a complex problem is approached by first turning to a solvable but necessarily simpler problem, followed by dimensional analysis, experimentation, and data correlation. If we look around in other fields of science and engineering, we find the same pattern of investigation as discussed here for fluid mechanics. This problem-solving methodology offers a place to start when we face a new and challenging problem.

10.8 Problems

1. Under what flow conditions are the Newtonian constitutive equation (Equation 10.2) and Newton's law of viscosity (Equation 10.1) equivalent?

2. Design a flow domain for numerical analysis on which to calculate the results from Hamel flow (see Figure 7.47). Indicate the boundary conditions on every boundary. Indicate regions where the mesh should be refined to improve accuracy.

3. Figure 10.3 is a proposed flow domain for the numerical evaluation of pressure-driven flow through a tube. Why has a section been added before the beginning of the test section?

4. A flow domain for the analysis of the steady flow through a two-dimensional contraction is shown in Figure 10.15. The flow rate is constant, the fluid is incompressible, and the fluid exits into air. What are the boundary conditions for this flow?
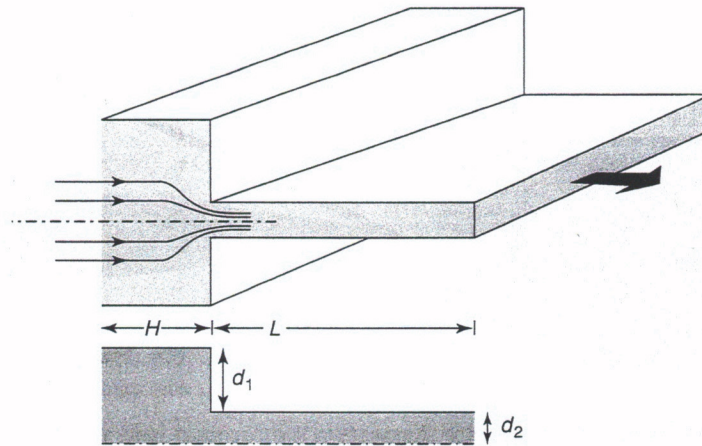

Figure 10.15   A flow domain for numerical evaluation of a flow (Problem 4).
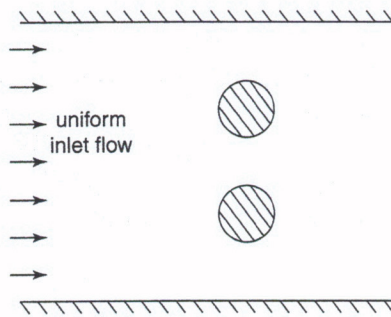
Figure 10.16 Flow field for Problem 5.

5. For the two-dimensional flow domain in Figure 10.16, indicate the regions where viscous effects are important and indicate the regions where inertial effects are expected to be important. The flow is rapid (i.e., Re is high) and the fluid is incompressible. The hashed regions are solid walls or obstacles.

6. For a pipe with laminar flow established, an increase in flow rate eventually causes the flow to become unstable and for turbulent flow to appear. Why is the flow unstable? Why is it that we cannot achieve arbitrarily high flow rates with laminar flow?

7. What are the $K-\epsilon$ models of turbulence? To answer this, research information in the literature.

8. What is the physical origin of the stresses described by the $\underline{\underline{\tilde{\tau}}}^{\text{turb}}$ term in the fluctuation-averaged Navier-Stokes equation for turbulent flow?

9. What is circulation?

10. Calculate the circulation around the center point of the free-vortex flow given in Equation 10.40.

11. Sketch the calculated flow field around an airfoil if too little and too much circulation are added. Sketch how the flow field around an airfoil should look if the correct amount of circulation (produced by the viscous flow in the boundary layer) is included in the model. What is the characteristic of the streamlines that shows that the correct amount of circulation has been added?

12. For the gradually contracting flow in Figure 10.17, where is the pressure highest? Indicate your answer on the a sketch of the flow. Explain.
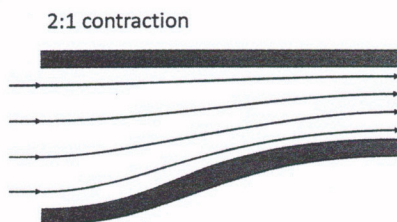
2:1 contraction

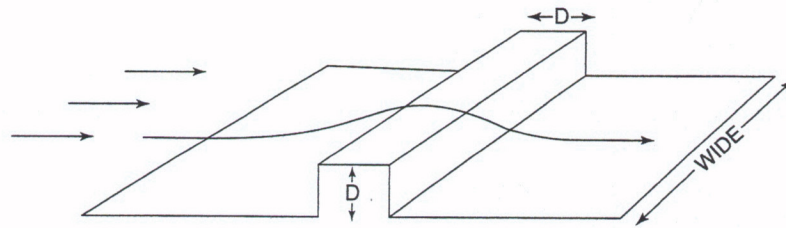Figure 10.17 A flow with streamline curvature (Problem 12).

Figure 10.18   Flow over a bump (Problem 16).

13. Calculate the pressure distribution in a spiral-vortex tank. The velocity field is given in Equation 10.40.

14. Using a numerical simulator [5, 27], calculate the velocity field for Poiseuille flow in a tube. Assume axisymmetric steady flow of an incompressible Newtonian fluid. Compare your result for the velocity field with the analytical solution obtained in Chapter 7.

15. Using a numerical simulator [5, 27], calculate the velocity field for flow in a 2:1 axisymmetric contraction. Assume two-dimensional steady flow of an incompressible Newtonian fluid. Choose a steady flow rate in the creeping-flow regime. What is the highest Reynolds number you can simulate with the available code?

16. Using a numerical simulator [5, 27], calculate the velocity field for flow over the bump shown in Figure 10.18. Assume two-dimensional steady flow of an incompressible Newtonian fluid. Make your calculations in the creeping-flow regime.

17. Using a numerical simulator [5, 27], calculate the velocity field for axial unidirectional flow in the void space around three tubes mounted inside a cylindrical shell (Figure 10.19). Assume steady flow of an incompressible Newtonian fluid. Make your calculations in the creeping-flow regime. Review the mesh in Figure 10.2.

18. Using a numerical simulator [5, 27], calculate the drag coefficient as a function of the Reynolds number for flow around a sphere. The flow is solved most easily in axisymmetric mode with a flow domain resembling that in Figure 10.20. Compare your results with experimental data.

19. Derive the mechanical energy balance for isothermal, compressible flow, Equation 10.97 [55].
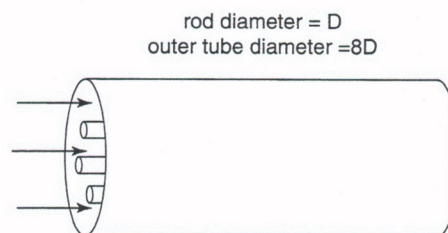
rod diameter = D
outer tube diameter =8D

Figure 10.19   Shell-and-tube heat exchangers have flow in the outer region similar to that shown here (Problem 17).
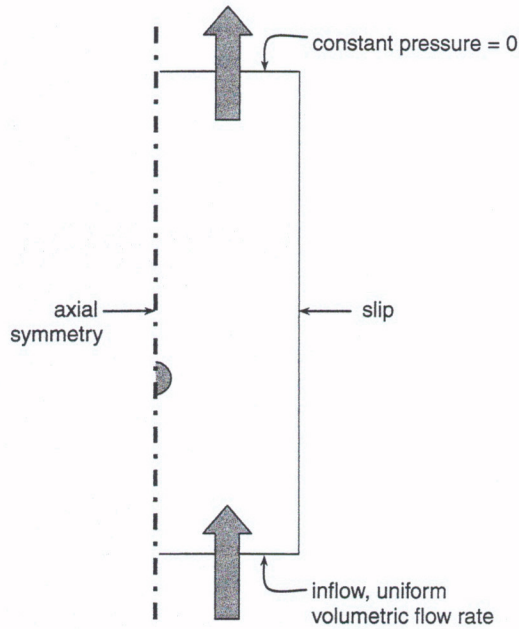
Figure 10.20 The steady, noncreeping flow around a sphere may be calculated using finite-element analysis or other numerical techniques. A reasonable formulation of the problem is to use the flow domain and boundary conditions shown here. The flow domain is chosen to be longer after the sphere to capture the wake structure expected to form there. The mesh chosen for the calculation should be refined near the sphere (Problem 18).

20. What is the speed of sound in room-temperature air? What is the speed of sound in air at $-20°C$?
21. What causes a shock wave? Investigate in the literature and provide citations.
22. How do the Navier-Stokes equations change if we want to study magnetohydrodynamics?
23. How do the Navier-Stokes equations change if we want to study flows with a rotating reference frame?
24. What is a Taylor column? Investigate in the literature and provide citations.
25. What is the Coriolis force? Investigate in the literature and provide citations.
26. Describe a tornado in fluid-mechanics terms.
27. Explain a boundary layer in terms suitable for an elementary-school student.
28. Explain the use of dimensionless numbers in fluid mechanics.
29. Discuss the utility of dividing flows into internal and external flows.
30. What are the forces accounted for in the Navier-Stokes equation? Which forces dominate? Explain.