Algebraic Details

(heat conduction in rect slab w/ Newton's Law of Cooling BC's)

BC2:

$$C_1\left(1+\frac{Bh_2}{k}\right) = h_2 C_2 - h_2 T_{b_2}$$

↑ SUBSTITUTE FROM BC1

$$\left(T_{b_1}-C_2\right)\left(1+\frac{Bh_2}{k}\right) = \frac{h_2 C_2}{h_1} - \frac{h_2 T_{b_2}}{h_1}$$

$$T_{b_1}\left(1+\frac{Bh_2}{k}\right) + \frac{h_2}{h_1} T_{b_2} = C_2\left(\frac{h_2}{h_1}+1+\frac{Bh_2}{k}\right)$$

$$= C_2 h_2\left[\frac{1}{h_1}+\frac{1}{h_2}+\frac{B}{k}\right]$$

$$C_2 = \frac{T_{b_1}\left(\frac{1}{h_2}+\frac{B}{k}\right) + \frac{1}{h_1} T_{b_2}}{\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}}$$

$$c_1 = h_1 (T_{b_1} - c_2)$$

$$\frac{c_1}{h_1} = T_{b_1} - \frac{T_{b_1}\left(\frac{1}{h_2} + \frac{B}{k}\right) + \frac{1}{h_1} T_{b_2}}{\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}}$$

$$= \frac{\frac{T_{b_1}}{h_1} + \cancel{\frac{T_{b_1} B}{k}} + \frac{T_{b_1}}{h_2} - \frac{T_{b_1}}{h_2} - \cancel{\frac{T_{b_1} B}{k}} - \frac{T_{b_2}}{h_1}}{\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}}$$

$$\boxed{c_1 = \frac{T_{b_1} - T_{b_2}}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}}$$

$$T = -\frac{q}{k}x + C_2 \qquad \text{let } \alpha = \frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}$$

$$T = -\frac{(T_{b_1} - T_{b_2})}{\alpha k}x + \frac{1}{\alpha}\left[\frac{T_{b_1}}{h_2} + \frac{T_{b_1}B}{k} + \frac{T_{b_2}}{h_1}\right]$$

SUBTRACT BOTH SIDES FROM T_{b_1}

$$T_{b_1} - T = \frac{(T_{b_1} - T_{b_2})}{\alpha k}x + T_{b_1} - \frac{\left[\frac{T_{b_1}}{h_2} + \frac{T_{b_1}B}{k} + \frac{T_{b_2}}{h_1}\right]}{\alpha}$$

$$\alpha(T_{b_1} - T) = \frac{(T_{b_1} - T_{b_2})}{k}x + \frac{T_{b_1}}{h_1} + \cancel{\frac{T_{b_1}B}{k}} + \cancel{\frac{T_{b_1}}{h_2}} - \cancel{\frac{T_{b_1}}{h_2}} - \cancel{\frac{T_{b_1}B}{k}} - \frac{T_{b_2}}{h_1}$$

$$= (T_{b_1} - T_{b_2})\left[\frac{x}{k} + \frac{1}{h_1}\right]$$

$$\boxed{\left(\frac{T_{b_1} - T}{T_{b_1} - T_{b_2}}\right) = \frac{\frac{x}{k} + \frac{1}{h_1}}{\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}}}$$