FINAL EXAM SOLN
CM3110
19 DEC 2007

1. (a) Re, Pr

(b) Gr, Pr

(c) Hagen Poiseuille eqn:

$$v_{z,av} = \frac{Q}{\pi R^2} = \frac{\Delta P D^2}{32 \mu L}$$

2.9-11 Geankoplis p85 (4th Ed)

or

$$f = \frac{16}{Re}$$

(d) The Moody Chart p94 Geankoplis (4th Ed)

Dinn (lec 14)

$$f = 0.079\, Re^{-0.25} \qquad 4000 \le Re \le 10^5$$

or

$$\frac{1}{\sqrt{f}} = 4.0 \log Re\sqrt{f} - 0.4 \qquad Re \geq 4000$$

or

Colebrook correlation w/ $k = 0$

$$\frac{1}{\sqrt{f}} = -4.0 \log\left(\frac{k}{D} + \frac{4.67}{Re\sqrt{f}}\right) + 2.28$$

(lecture 14 or Denn)

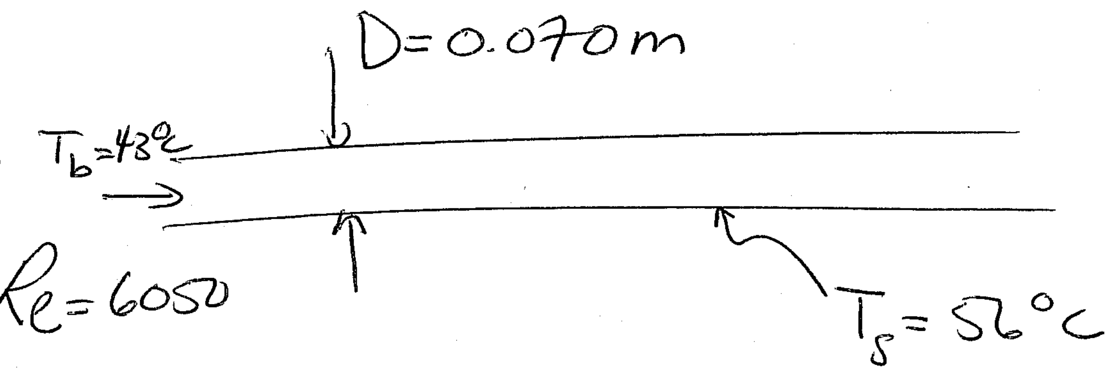
$$V = Q/\pi R^2$$

$$Re = \frac{\rho V D}{\mu}$$

$$f = \frac{\Delta P \pi R^2}{2\pi R \Delta L} \quad \frac{1}{\frac{1}{2}\rho V^2}$$

2.10-4 Geankoplis 4th Ed (p91)

2.



$$Q = h A (T_s - T_b)$$

from correlation for turbulent water flow

$$\frac{h_c D}{k} = 0.027 Re^{0.8} Pr^{1/3} \cancel{\left(\frac{\mu_b}{\mu_w}\right)^{0.14}}$$

temp effects not important

$$= 0.027 (6050)^{0.8} (4.05)^{1/3}$$

$$= (0.027)(1060.24)(1.593987)$$

$$\frac{h_c D}{k} = 45.6302$$

$$h_L = \left(\frac{0.6283\,W}{mK}\right)\left(\frac{1}{.070m}\right)\left(45.6302\right)$$

$$= 409.56$$

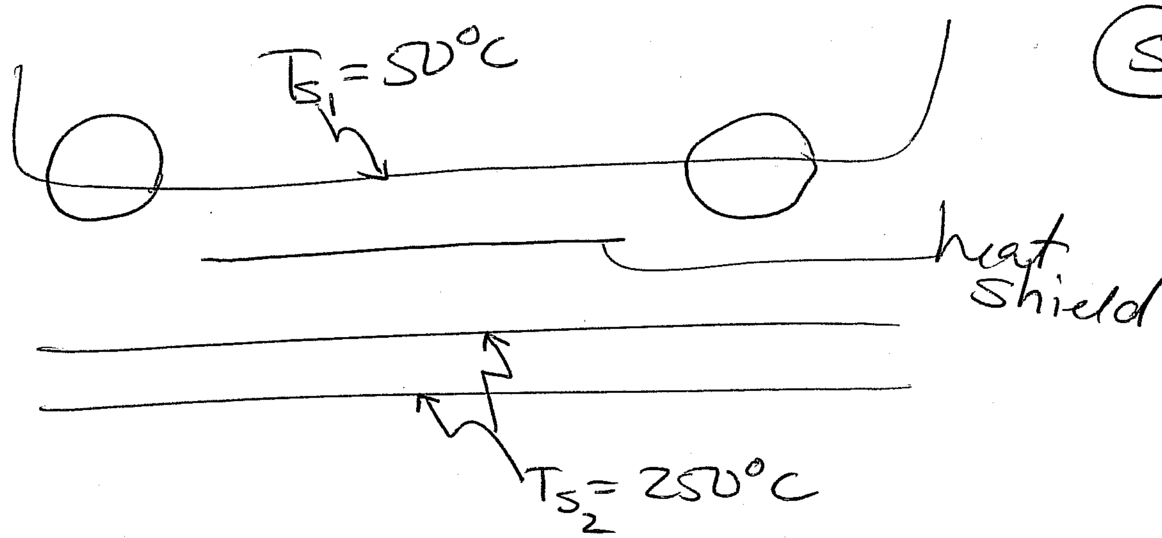
$$\boxed{410\ \frac{W}{m^2K}}$$

2 SIG FIGS
(diameter is
known to
2 sig figs
only)

//

3

5



$\frac{q}{A} = ?$

<u>radiation</u> (lecture 20)

$$\frac{q}{A} = \frac{1}{N+1} \frac{\sigma (T_1^4 - T_3^4)}{\frac{2}{\varepsilon} - 1}$$

$N = 1$ (# heat shields)

$\varepsilon = \alpha = 0.79$

$T_1 = (250 + 273.16)K = 523.16K$

$T_3 = (50 + 273.16)K = 323.16K$

$\sigma = 5.676 \times 10^{-8} \frac{W}{m^2K^4}$

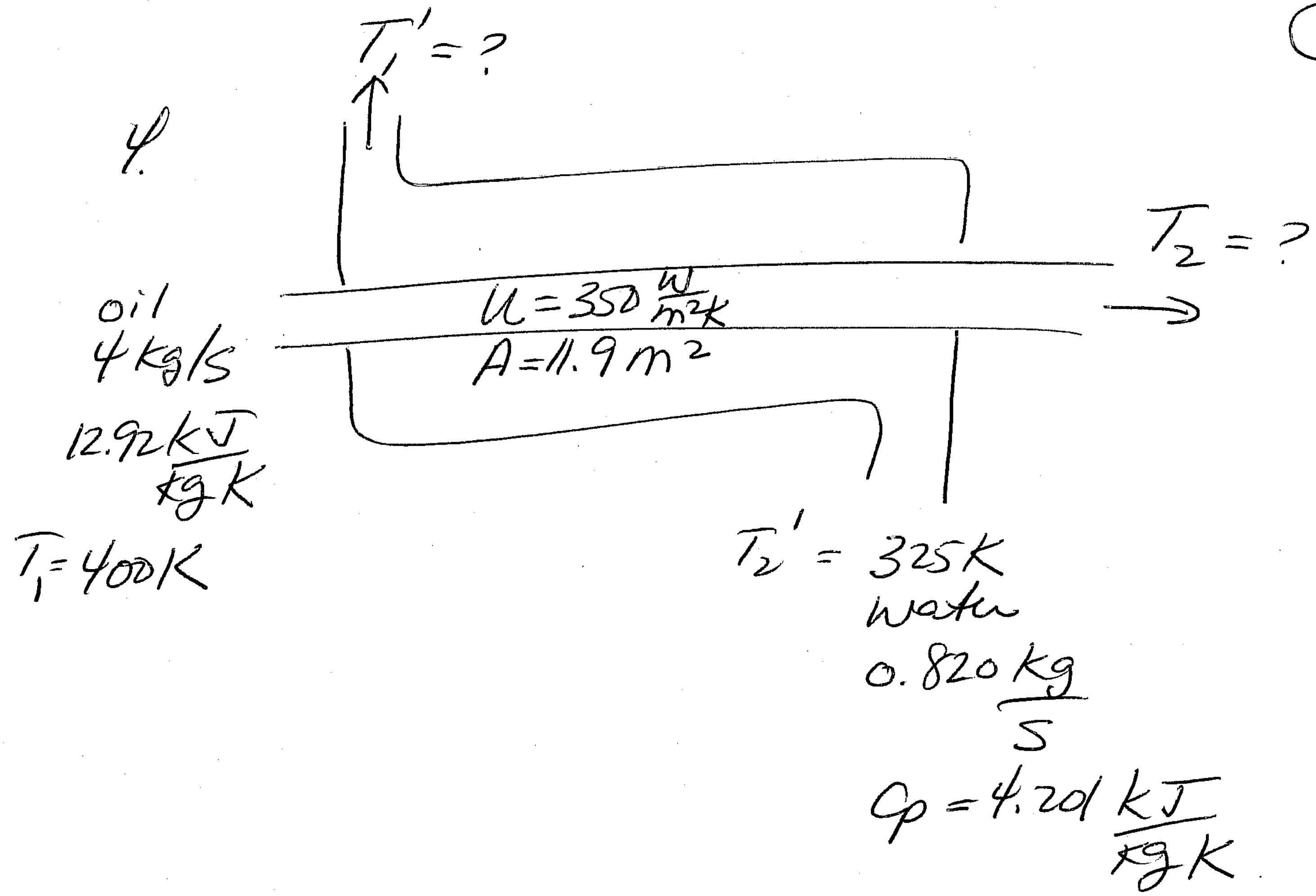
$$\frac{q}{A} = \frac{\left(\frac{1}{2}\right)\left(5.676\times10^{-8}\frac{W}{m^2K^4}\right)\left[(523.16K)^4 - (323.16K)^4\right]}{\left(\frac{2}{0.79} - 1\right)}$$

$$= 1185.928\ \frac{W}{m^2}$$

$$= \boxed{1200\ \frac{W}{m^2}}$$

2 sig figs on Emissivity

4.



Inlet conditions known $\Rightarrow$
heat exchanger
effectiveness

oil:

$$mC_p = \left(\frac{4 kg}{s}\right)\left(\frac{12.92 kJ}{kg K}\right) = 51.68 \frac{kJ}{sK}$$

water

$$mC_p = \left(\frac{0.820 kg}{s}\right)\left(4.201 \frac{kJ}{kg K}\right) = 3.4448 \frac{kJ}{sK}$$

$\Rightarrow$ water is minimum fluid.

$$\frac{C_{min}}{C_{max}} = \frac{3.4448}{51.68} = 0.0667 \cong 0.1$$

$$NTU = \frac{UA}{C_{min}} = \frac{\left(350 \frac{W}{\cancel{m^2K}}\right)\left(11.9\ \cancel{m^2}\right)}{\left(3.4448\ \frac{\cancel{kW}}{K}\right)\left(\frac{10^3\ W}{\cancel{kW}}\right)}$$

$$NTU = 1.209 \cong 1.2$$

read chart:

$\varepsilon = 0.67$ (Fig 4.9-7 p 299 Geankoplis.

$$Q = \varepsilon\,(\dot{m}C_p)_{min}\,(T_{h_1} - T_{c_i})$$

$$= (0.67)\left(3.4448\ \frac{kW}{\cancel{K}}\right)(400-325)\cancel{K}$$

$= 173.1$ kW OR $\boxed{200\ kW}$

$= \boxed{170\ kW}$ } reading graph = 1 SIG FIG

Exit oil temp:

E-BAL:

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{sh}}$$

$$Q_{in} = \Delta H = mC_p\,(T_{out} - T_{in})$$

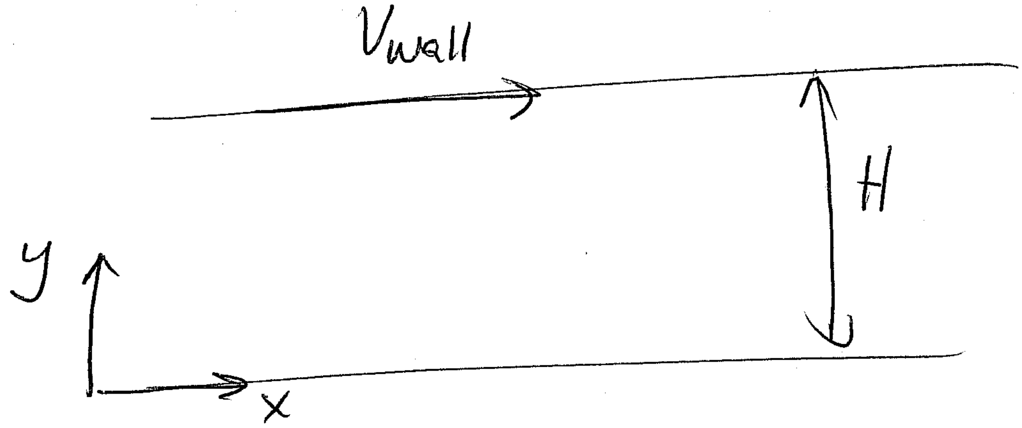
$$\frac{Q}{(mC_p)_{oil}} = T_{out} - T_{in}$$

$$T_{out} = \frac{(-173.1\ kW)}{\left(51.68\ \frac{kW}{K}\right)} + 400K$$

$$= 396.65$$

$$= \boxed{397K}$$

5.



① $\underline{v} = \begin{pmatrix} v_x \\ 0 \\ 0 \end{pmatrix}$ ② steady ③ $\rho = $ const.

Mass bal: $\nabla \cdot \underline{V} = 0$

$$\frac{\partial v_x}{\partial x} + \cancel{\frac{\partial v_y}{\partial y}} + \cancel{\frac{\partial v_z}{\partial z}} = 0$$

$$\boxed{\frac{\partial v_x}{\partial x} = 0}$$

momentum bal: see sheet.

y component $\frac{\partial P}{\partial y} = 0$

z component $\frac{\partial P}{\partial z} = 0$

The Equation of Continuity and the Equation of Motion in Cartesian, cylindrical, and spherical coordinates

CM3110 Fall 2007 Faith A. Morrison

Continuity Equation, Cartesian coordinates

steady

$$\cancel{\frac{\partial \rho}{\partial t}} + \left(v_x \cancel{\frac{\partial \rho}{\partial x}} + v_y \cancel{\frac{\partial \rho}{\partial y}} + v_z \cancel{\frac{\partial \rho}{\partial z}} \right) + \rho \left(\frac{\partial v_x}{\partial x} + \cancel{\frac{\partial v_y}{\partial y}} + \cancel{\frac{\partial v_z}{\partial z}} \right) = 0$$

$\rho = const$

$v_y = v_z = 0$

$\Rightarrow \boxed{\frac{\partial v_x}{\partial x} = 0}$

Continuity Equation, cylindrical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial (\rho r v_r)}{\partial r} + \frac{1}{r}\frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

Continuity Equation, spherical coordinates

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial (\rho r^2 v_r)}{\partial r} + \frac{1}{r \sin\theta}\frac{\partial (\rho v_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial (\rho v_\phi)}{\partial \phi} = 0$$

Equation of Motion for an incompressible fluid, 3 components in Cartesian coordinates

$$\begin{aligned}
\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) &= -\frac{\partial P}{\partial x} - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \rho g_x \\
\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) &= -\frac{\partial P}{\partial y} - \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \rho g_y \\
\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) &= -\frac{\partial P}{\partial z} - \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z
\end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in cylindrical coordinates

$$\begin{aligned}
\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) &= -\frac{\partial P}{\partial r} - \left(\frac{1}{r}\frac{\partial (r\tau_{rr})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r \\
\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) &= -\frac{1}{r}\frac{\partial P}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial (r^2\tau_{r\theta})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta \\
\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) &= -\frac{\partial P}{\partial z} - \left(\frac{1}{r}\frac{\partial (r\tau_{rz})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z
\end{aligned}$$

Equation of Motion for an incompressible fluid, 3 components in spherical coordinates

$$\begin{aligned}
&\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) \\
&= -\frac{\partial P}{\partial r} - \left(\frac{1}{r^2}\frac{\partial (r^2\tau_{rr})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\tau_{r\theta}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right) + \rho g_r \\
&\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right) \\
&= -\frac{1}{r}\frac{\partial P}{\partial \theta} - \left(\frac{1}{r^2}\frac{\partial (r^2\tau_{r\theta})}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\tau_{\theta\theta}\sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{(\cot\theta)\tau_{\phi\phi}}{r}\right) + \rho g_\theta \\
&\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\phi v_\theta\cot\theta}{r}\right) \\
&= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi} - \left(\frac{1}{r^2}\frac{\partial (r^2\tau_{r\phi})}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\phi}}{r} + \frac{(2\cot\theta)\tau_{\theta\phi}}{r}\right) + \rho g_\phi
\end{aligned}$$

=0 by mass bal

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation) 3 components in Cartesian coordinates

Steady $\qquad v_x = v_y = 0 \qquad$ wide

$$\begin{aligned}
\rho\left(\cancel{\frac{\partial v_x}{\partial t}} + v_x\frac{\partial v_x}{\partial x} + \cancel{v_y}\frac{\partial v_x}{\partial y} + \cancel{v_z}\frac{\partial v_x}{\partial z}\right) &= -\frac{\partial P}{\partial x} + \mu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \frac{\partial^2 v_x}{\partial y^2} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}\right) + \cancel{\rho g_x} \\
\rho\left(\cancel{\frac{\partial v_y}{\partial t}} + v_x\cancel{\frac{\partial v_y}{\partial x}} + \cancel{v_y}\frac{\partial v_y}{\partial y} + \cancel{v_z}\frac{\partial v_y}{\partial z}\right) &= -\frac{\partial P}{\partial y} + \mu\left(\cancel{\frac{\partial^2 v_y}{\partial x^2}} + \cancel{\frac{\partial^2 v_y}{\partial y^2}} + \cancel{\frac{\partial^2 v_y}{\partial z^2}}\right) + \cancel{\rho g_y} \\
\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + v_x\cancel{\frac{\partial v_z}{\partial x}} + \cancel{v_y}\frac{\partial v_z}{\partial y} + \cancel{v_z}\frac{\partial v_z}{\partial z}\right) &= -\frac{\partial P}{\partial z} + \mu\left(\cancel{\frac{\partial^2 v_z}{\partial x^2}} + \cancel{\frac{\partial^2 v_z}{\partial y^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right) + \cancel{\rho g_z}
\end{aligned}$$

$\underline{g}$ neglected

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in cylindrical coordinates

$$\begin{aligned}
\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) &= -\frac{\partial P}{\partial r} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right) + \rho g_r \\
\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) &= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right) + \rho g_\theta \\
\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) &= -\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z
\end{aligned}$$

Equation of Motion for incompressible, Newtonian fluid (Navier-Stokes equation), 3 components in spherical coordinates

$$\begin{aligned}
&\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2+v_\phi^2}{r}\right) \\
&= -\frac{\partial P}{\partial r} + \mu\left(\nabla^2 v_r - \frac{2v_r}{r^2} - \frac{2}{r^2}\frac{v_\theta}{\partial \theta} - \frac{2v_\theta\cot\theta}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right) + \rho g_r \\
&\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right) \\
&= -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left(\nabla^2 v_\theta + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial \phi}\right) + \rho g_\theta \\
&\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi}{r} + \frac{v_\phi v_\theta\cot\theta}{r}\right) \\
&= -\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi} + \mu\left(\nabla^2 v_\phi - \frac{v_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial \phi}\right) + \rho g_\phi
\end{aligned}$$

where, in these equations, $\nabla \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial \phi^2}\right)$.

Reference: R. B. Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, Wiley: NY, 1960.

X-component:

$$0 = \underbrace{-\frac{\partial P}{\partial x}}_{\frac{\Delta P}{L}} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

$$\underbrace{-\frac{\Delta P}{\mu L}}_{\text{constant}} = \frac{d^2 v_x}{dy^2}$$

integrate: $\frac{dv_x}{dy} = \left(-\frac{\Delta P}{\mu L}\right) y + c_1$

$$\boxed{v_x = -\frac{\Delta P}{\mu L} \frac{y^2}{2} + c_1 y + c_2}$$

BC: $y = 0 \quad v_x = 0 \Rightarrow \boxed{c_2 = 0}$

$y = H \quad v_x = V_{wall}$

$$V_{wall} = -\frac{\Delta P}{2 \mu L} H^2 + c_1 H$$

$$c_1 = \frac{1}{H}\left(V_{wall} + \frac{\Delta P H^2}{2\mu L}\right)$$

$$v_x = -\frac{\Delta P}{2\mu L} y^2 + \frac{y}{H}\left(V_{wall} + \frac{\Delta P H^2}{2\mu L}\right)$$

or, simplifying

$$\boxed{v_x = \frac{y V_{wall}}{H} + \frac{\Delta P H^2}{2\mu L}\left(\frac{y}{H} - \frac{y^2}{H^2}\right)}$$