# CM3110
# Transport I
# Part I: Fluid Mechanics

*Topic 1: Microscopic Balances*

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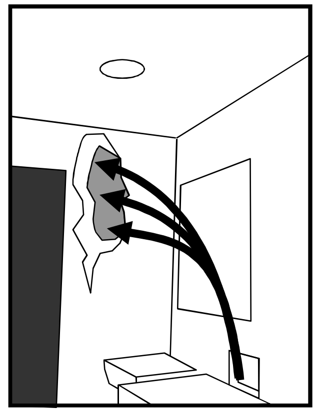
*A problem from real life:*

The hose connecting the city water supply to the washing machine in a home burst while the homeowner was away Water spilled out of the ½ *in* pipe for 48 hours before the problem was noticed by a neighbor and the water was cut off.

**How much water sprayed into the house over the 2-day period?**

The water utility reports that the water pressure supplied to the house was approximately 60 *psig*.

**Home flood:**
the cold-water feed to a washing machine burst and was unattended for two days






**Discussion:**

How do we calculate the total amount of water spilled?

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What determines flow rate through a pipe?




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How do we calculate the total amount of water spilled?

What determines flow rate through a pipe?

What information do we need about the system to calculate the amount of water spilled over two days?

## House flood problem ⇒

## Solution Strategy:

- Apply the laws of physics to the situation
- Calculate the velocity field in the pipe
  (will be function of pressure)
- Calculate the flow rate from the velocity field
  (as a function of pressure)
- Calculate the total amount of water spilled

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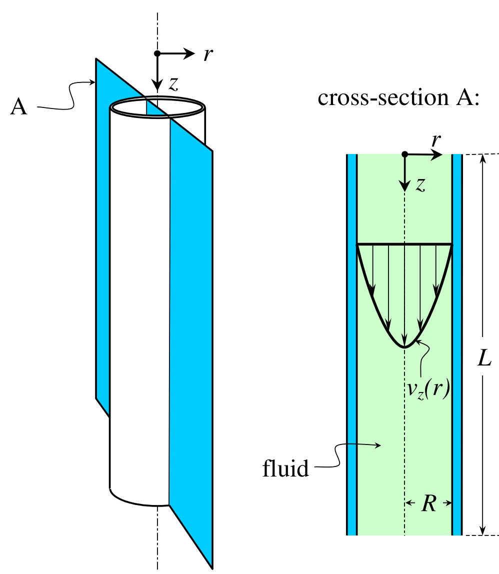
## The Situation:

Steady flow of water in a pipe

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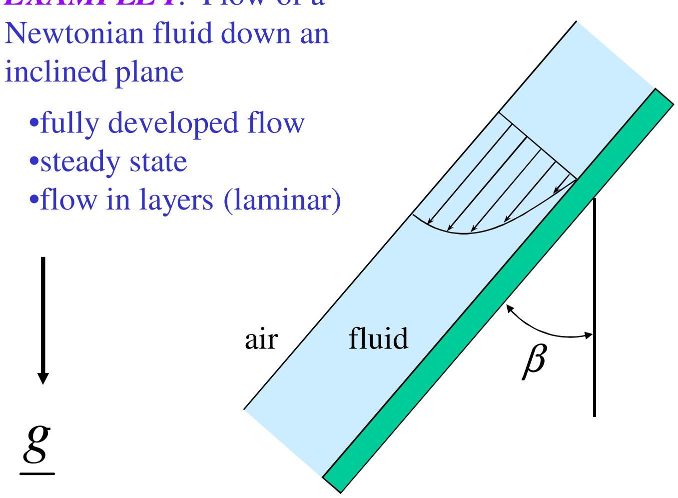
**Next step: perform balances on flow in a tube**

Because flow in a tube is a bit complicated to do as a first problem (because of the curves), let's consider a somewhat simpler problem first.




***EXAMPLE I***: Flow of a Newtonian fluid down an inclined plane

- •fully developed flow
- •steady state
- •flow in layers (laminar)



## The Laws of Physics

Mass is conserved
Momentum is conserved
Energy is conserved




## Physics I: Mechanics

**Mass is conserved**

- **This was not an issue in mechanics because we study bodies and the mass of the body does not change**
- **Now we have to worry about it because we study fluids**

**•Momentum is conserved**

- **Newton's 2nd law:**

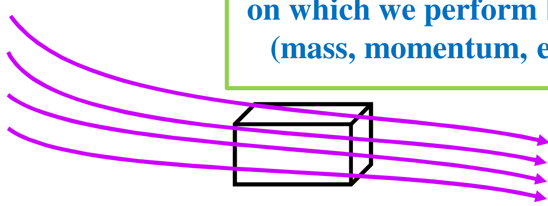
$$\sum_{on\,body} \underline{F} = m\underline{a}$$

- **The "body" is ill-defined in flow**

**•Energy is conserved (similar issues)**

## Control Volume

**A chosen volume in a flow on which we perform balances (mass, momentum, energy)**



**•Shape, size are arbitrary; choose to be convenient**

**•Because we are now balancing on *control volumes* instead of on *bodies*, the laws of physics are written differently**




**Mass balance, flowing system (open system; control volume):**

$$\left\{\begin{matrix} net\ mass \\ flowing\ in \end{matrix}\right\} = \left\{\begin{matrix} \cancel{rate\ of} \\ \cancel{accumulation} \\ \cancel{of\ mass} \end{matrix}\right\}$$

$$\sum_{in} - \sum_{out} \qquad\qquad \textit{steady state}$$

**Momentum balance, flowing system (open system; control volume):**

$$\left\{\begin{matrix}\textit{sum of forces}\\ \textit{acting } \underline{\textit{on}} \textit{ control vol}\end{matrix}\right\} + \underbrace{\left\{\begin{matrix}\textit{net momentum}\\ \textit{flowing in}\end{matrix}\right\}}_{\sum_{in} - \sum_{out}} = \underbrace{\left\{\begin{matrix}\textit{rate of}\\ \textit{accumulation}\\ \textit{of momentum}\end{matrix}\right\}}_{\textit{steady state}}$$

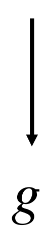
$$\sum_i F_{on_i} + \sum_i \left\{\begin{matrix}\textit{momentum}\\ \textit{flowing } \underline{\textit{in}}\\ \textit{in the streams}\end{matrix}\right\} - \sum_i \left\{\begin{matrix}\textit{momentum}\\ \textit{flowing } \underline{\textit{out}}\\ \textit{in the streams}\end{matrix}\right\} = 0$$

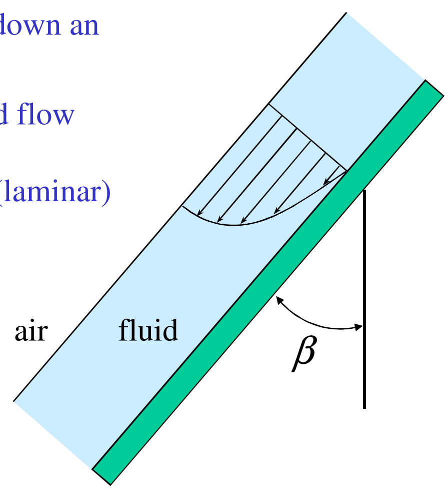
note that momentum is a vector quantity




***EXAMPLE I***: Flow of a Newtonian fluid down an inclined plane

- fully developed flow
- steady state
- flow in layers (laminar)



## Problem-Solving Procedure - fluid-mechanics problems

1. sketch system
2. choose coordinate system
3. choose a control volume
4. perform a mass balance
5. perform a momentum balance

   (will contain stress)
6. substitute in *Newton's law of viscosity,* e.g. $\tau_{yz} = -\mu \left( \frac{dv_z}{dy} \right)$
7. solve the <u>differential</u> equation
8. apply boundary conditions





$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz} = \begin{pmatrix} v_x \\ 0 \\ v_z \end{pmatrix}_{xyz}$$

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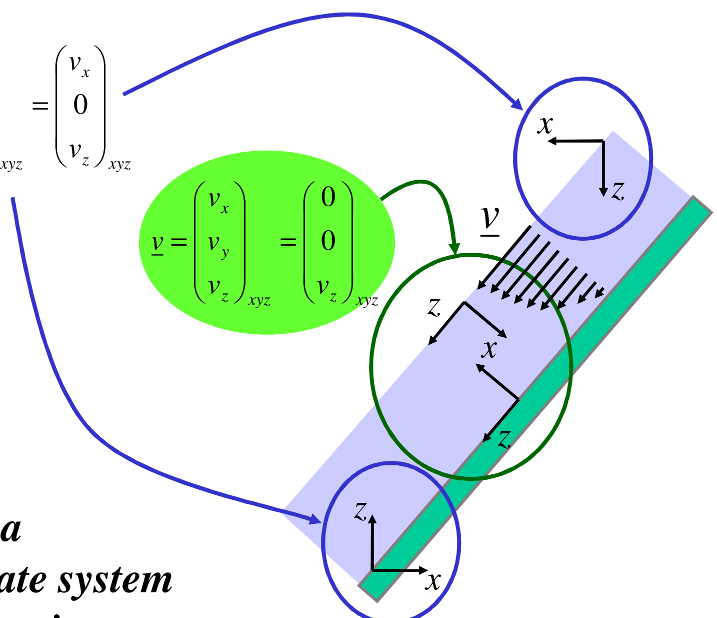
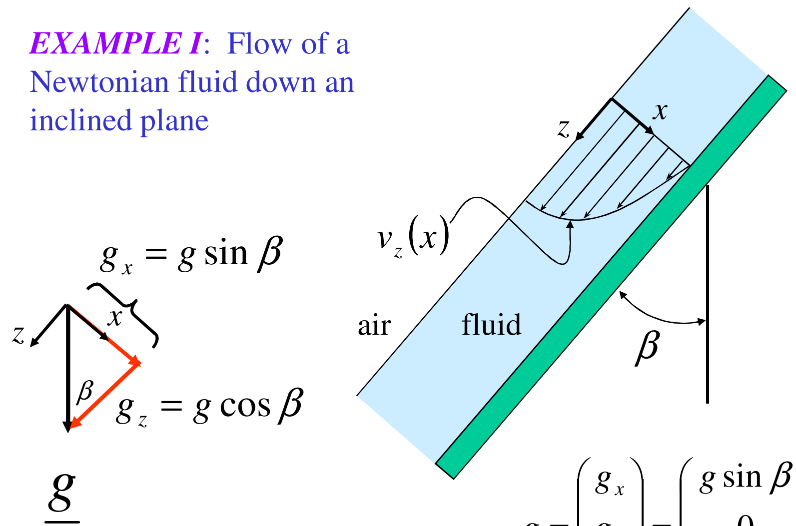



***Choose a coordinate system for convenience***

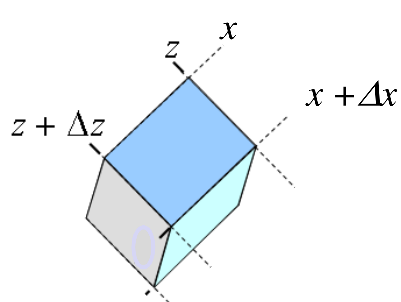
***EXAMPLE I***: Flow of a Newtonian fluid down an inclined plane




$$\underline{g} = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{pmatrix} g \sin\beta \\ 0 \\ g \cos\beta \end{pmatrix}$$




Choose a convenient control volume



Assumptions: (laminar flow down an incline, Newtonian)

1. no velocity in the *x*- or *y*-directions (laminar flow)
2. no shear stress at interface
3. no slip at wall
4. Newtonian fluid
5. steady state
6. well developed flow
7. no edge effects in *y*-direction (width)
8. constant density




## Flow down an Incline Plane

**Boundary conditions:**

$$x = 0 \quad \tau_{xz} = 0$$ -stress matches at boundary

$$x = H \quad v_z = 0$$ -no slip at the wall

**Solution:**

$$v_z(x) = \frac{\rho g \cos(\beta)}{2\mu}\left(H^2 - x^2\right)$$

# Engineering Quantities of Interest

average velocity

$$\langle v_z \rangle \equiv \frac{\int_0^W \int_0^\delta v_z \, dx \, dy}{\int_0^W \int_0^\delta dx \, dy}$$

$\delta$ is the height of the film

volumetric flow rate

$$Q = \int_0^W \int_0^\delta v_x \, dx \, dy = W\delta \langle v_z \rangle$$

z-component of force on the wall

$$F_z = \int_0^L \int_0^W \tau_{xz}\Big|_{x=\delta} dy \, dz$$

(The expressions are different in different coordinate systems)





## What is the shear stress as a function of position for this flow?

Newton's Law of Viscosity

$$\tau_{xz} = -\mu \frac{\partial v_z}{\partial x}$$

**We have solved for $v_z(x)$; we can now calculate the shear stress.**