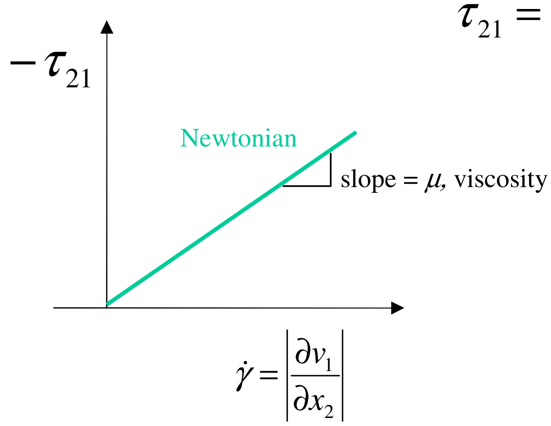
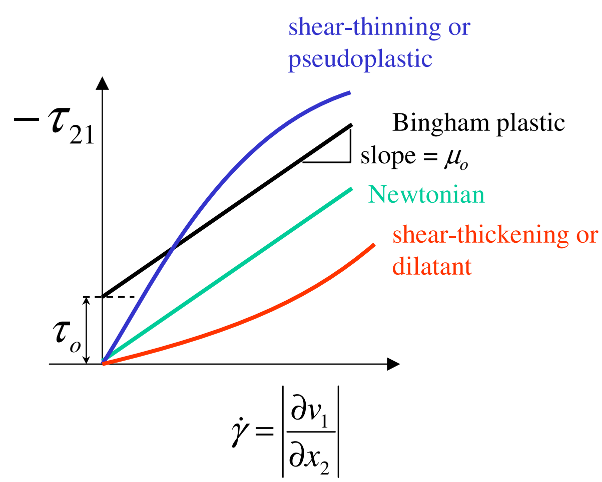
Newtonian Fluids

Newton's Law of Viscosity

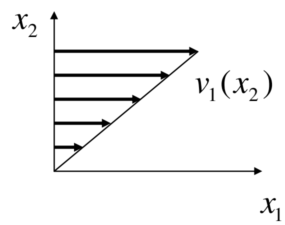
$$\tau_{21} = -\mu\left(\frac{dv_1}{dx_2}\right)$$



Non-Newtonian Fluids



What is the definition of viscosity for Non-Newtonian Fluids?

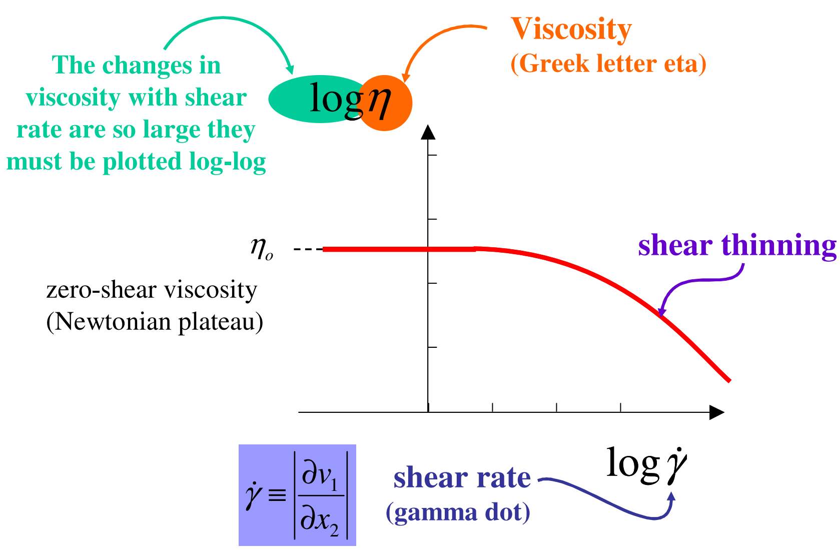


$$\dot{\gamma} = \left| \frac{\partial v_1}{\partial x_2} \right|$$

(NOTE on coordinate system: Viscosity definition is written for shear flow in x_1 direction and gradient in x_2 direction)

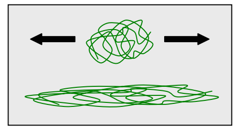
$$\eta \equiv \frac{-\tau_{21}}{\dot{\gamma}}$$

typical polymer behavior



$$\dot{\gamma} \equiv \left| \frac{\partial v_1}{\partial x_2} \right|$$

In addition, for many polymers there are shear-induced **NORMAL** (perpendicular) forces.

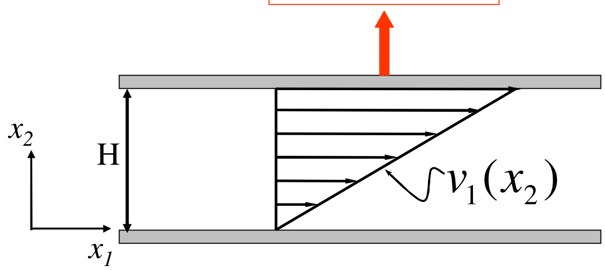


force on 2-surface in 2-direction

$$\int_A (\tau_{22} + P)\, dA$$

force on *y*-surface in *z*-direction

$$\int_A \tau_{21}\, dA$$



Power-Law Model (Ostwald-deWaele Model)

(does **not** model normal stresses)

$$\tau_{21} = -m \left| \frac{dv_1}{dx_2} \right|^{n-1} \frac{dv_1}{dx_2}$$

m or *K* = consistency index (m = μ for Newtonian)
n = power-law index (*n* = 1 for Newtonian)

$$\dot{\gamma} \equiv \left| \frac{\partial v_1}{\partial x_2} \right| = shear\ rate$$

What does the power-law model predict for viscosity?

$$\tau_{21} = -m\left|\frac{dv_1}{dx_2}\right|^{n-1}\frac{dv_1}{dx_2}$$

$$\eta \equiv \frac{-\tau_{21}}{\dot{\gamma}} = \frac{-\tau_{21}}{\left|\frac{dv_1}{dx_2}\right|} = m\left|\frac{dv_1}{dx_2}\right|^{n-1}$$

On a log-log plot, this would give a straight line:

$$\underbrace{\log\eta}_{Y} = \underbrace{\log m}_{B} + \underbrace{(n-1)}_{M}\underbrace{\log\left|\frac{dv_1}{dx_2}\right|}_{X}$$

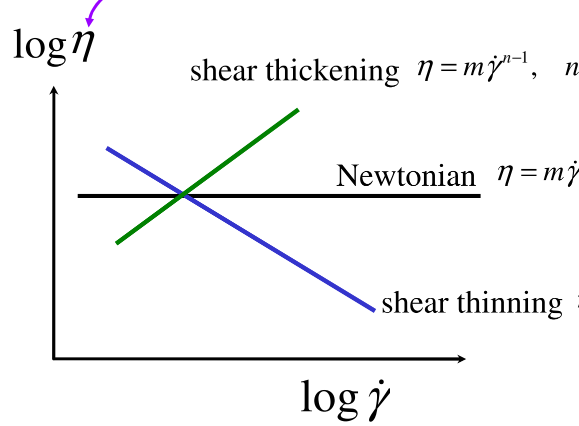
$$Y = B + MX$$

Power-Law Fluid

Non-Newtonian viscosity

$$\eta \equiv \frac{-\tau_{21}}{\dot{\gamma}}$$

$$\dot{\gamma} = \left|\frac{\partial v_1}{\partial x_2}\right|$$



shear thickening $\eta = m\dot{\gamma}^{n-1}, \quad n > 1$

Newtonian $\eta = m\dot{\gamma}^{n-1}, \quad n = 1$

shear thinning $\eta = m\dot{\gamma}^{n-1}, \quad n < 1$

Bingham Model (yield-stress fluid)

$$\tau_{21} = \begin{cases} 0 & |\tau_{21}| < \tau_o \\ -\mu_o \dfrac{dv_1}{dx_2} + \tau_o & |\tau_{21}| \geq \tau_o \end{cases}$$

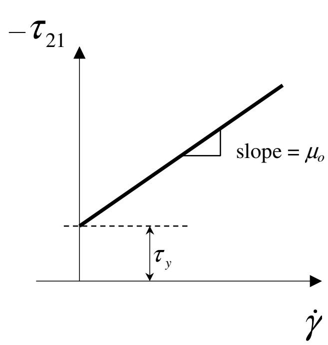
μ_o = viscosity parameter
τ_o = yield stress

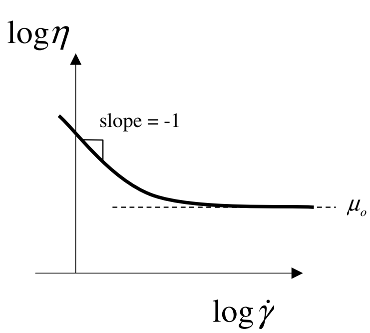
example: mayonnaise

There is no flow until the shear stress exceeds a critical value called the yield stress.

Bingham Plastic

$$\dot{\gamma} = \left| \frac{\partial v_1}{\partial x_2} \right|$$





where do we use the power-law expression?

e.g., Poiseuille flow in a tube:

Newtonian

$$\tau_{rz} = -\mu\left(\frac{dv_z}{dr}\right)$$

$$\tau_{rz} = \left(\frac{L\rho g + (P_o - P_L)}{2L}\right)r$$

non-Newtonian, power-law

$$\tau_{rz} = -m\left|\frac{dv_z}{dr}\right|^{n-1}\frac{dv_z}{dr}$$

$\Rightarrow$ **solve for $v_z(r)$**

1-direction = r
2-direction = z

Calculate the velocity field for
Poiseuille flow of a power-law fluid:

$$\tau_{rz} = -m\left|\frac{dv_z}{dr}\right|^{n-1}\frac{dv_z}{dr} = \left(\frac{L\rho g + (P_o - P_L)}{2L}\right)r$$

$$= -\frac{\partial v_z}{\partial r} \quad \forall r \qquad \equiv \alpha$$

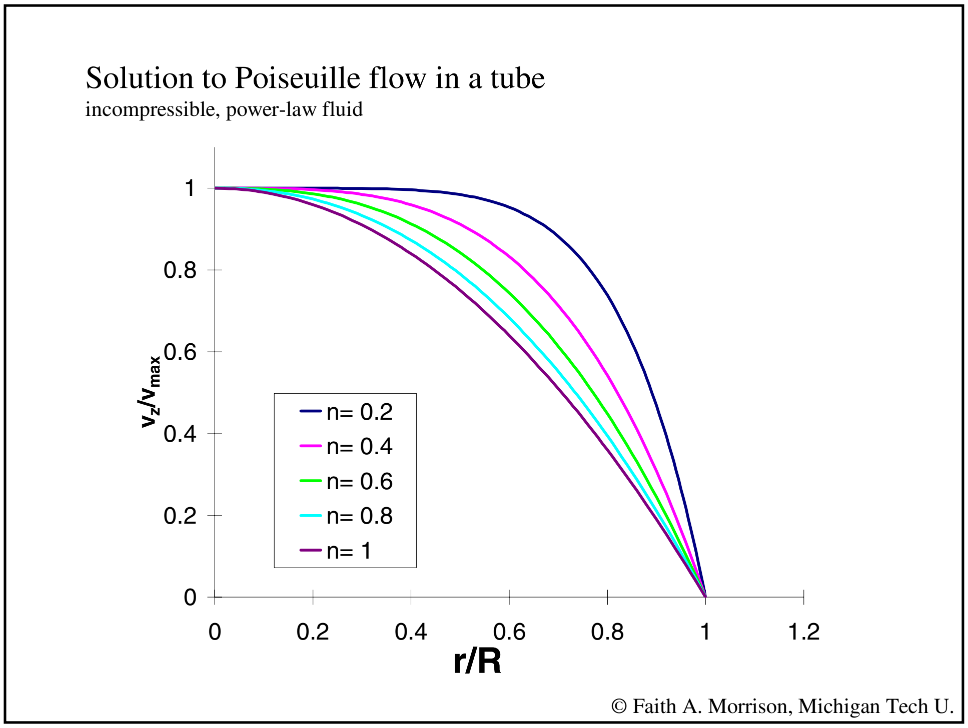
$$-m\left(-\frac{dv_z}{dr}\right)^{n-1}\frac{dv_z}{dr} = m\left(-\frac{dv_z}{dr}\right)^{n} = \alpha r$$

Solve for $v_z(r)$

Boundary Conditions: ?

Velocity field
Poiseuille flow of a power-law fluid:

$$v_z(r) = \left(\frac{R\left(L\rho g + P_o - P_L\right)}{2Lm} \right)^{\frac{1}{n}} \left(\frac{R}{\frac{1}{n}+1} \right) \left(1 - \left(\frac{r}{R} \right)^{\frac{1}{n}+1} \right)$$


Solution to Poiseuille flow in a tube
incompressible, power-law fluid
1
0.8
0.6
0.4
0.2
0
v_z/v_{max}
n= 0.2
n= 0.4
n= 0.6
n= 0.8
n= 1
0
0.2
0.4
0.6
0.8
1
1.2
r/R
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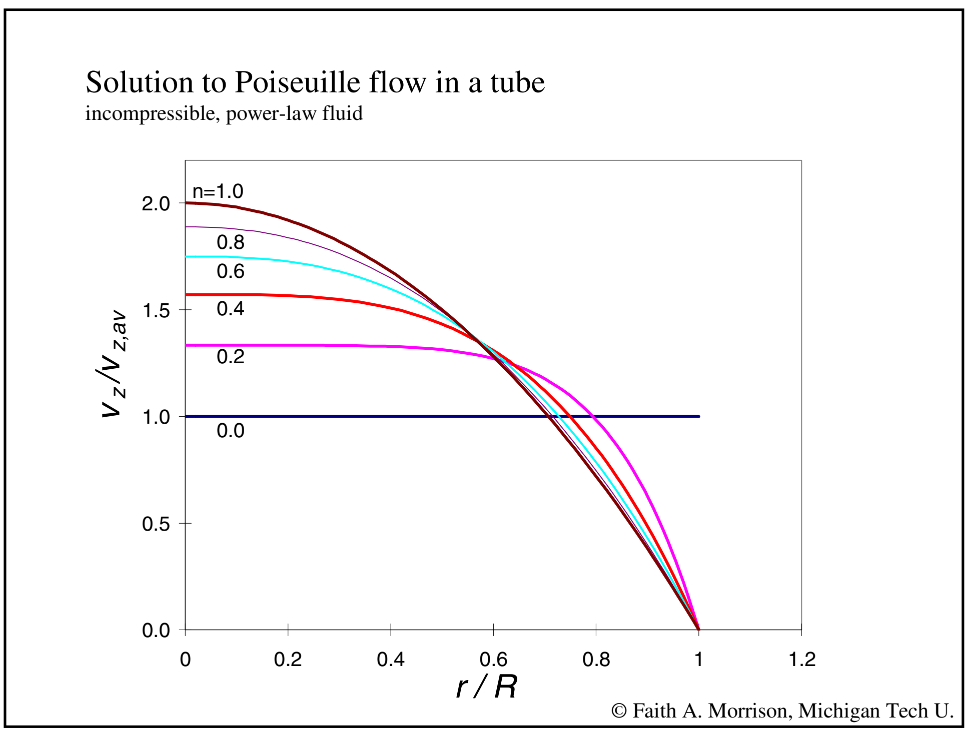
Solution to Poiseuille flow in a tube
incompressible, power-law fluid
n=1.0
0.8
0.6
0.4
0.2
0.0
2.0
1.5
1.0
0.5
0.0
$v_z/v_{z,av}$
0
0.2
0.4
0.6
0.8
1
1.2
r / R
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