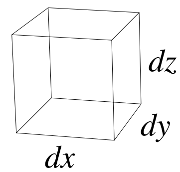
Microscopic Balances



We have been doing microscopic *shell* balances that are specific to whatever problem we are solving.

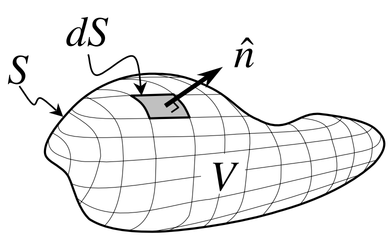
We seek equations for microscopic mass, momentum (and energy) balances that are *general*.

$\Rightarrow$ •equations must not depend on the choice of the control volume,

•equations must capture the appropriate balance

Continuity Equation

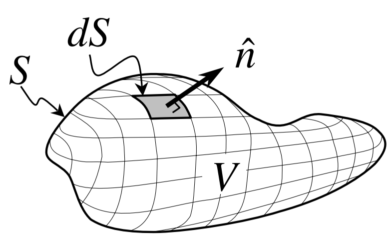
microscopic **mass** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*



$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

Gibbs notation: $\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho = -\rho (\nabla \cdot \underline{v})$

Equation of Motion



microscopic **momentum** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla P - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$$ **general fluid**

Gibbs notation: $$\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla P + \mu\nabla^2\underline{v} + \rho\underline{g}$$ **Newtonian fluid**

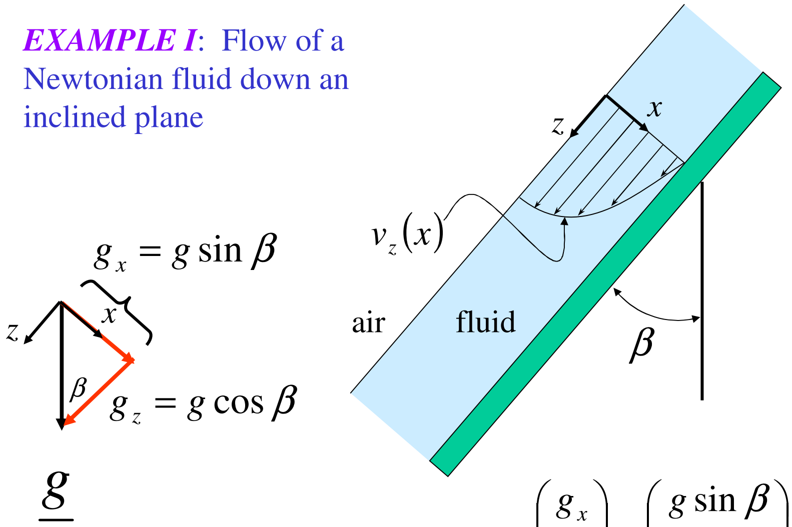
Navier-Stokes Equation

Problem-Solving Procedure - fluid-mechanics problems

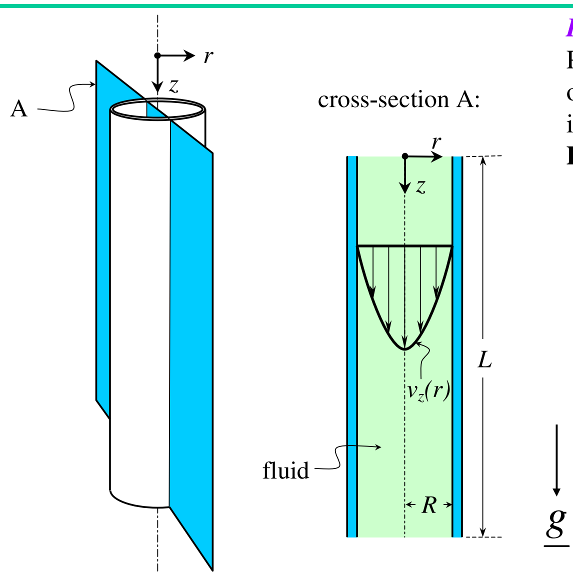
amended: when using the microscopic balances

1. sketch system
2. choose coordinate system
3. simplify the *continuity* equation (mass balance)
4. simplify the 3 components of the *equation of motion* (momentum balance) *(note that for a Newtonian fluid, the equation of motion is the Navier-Stokes equation)*
5. solve the differential equations for velocity and pressure (if applicable)
6. apply boundary conditions
7. calculate any engineering values of interest (flow rate, average velocity, force on wall)

EXAMPLE I: Flow of a Newtonian fluid down an inclined plane



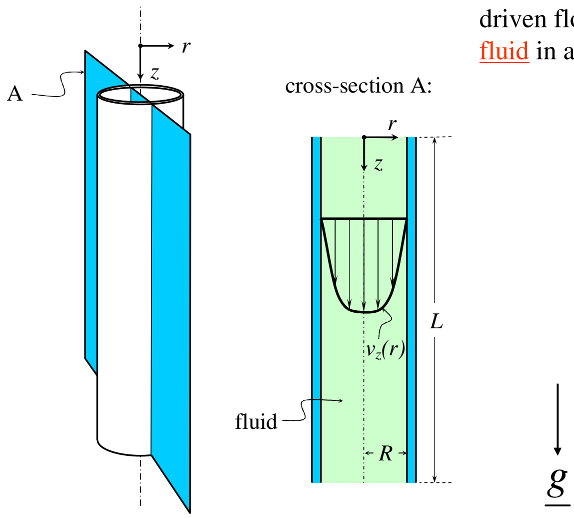
$$\underline{g} = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{pmatrix} g\sin\beta \\ 0 \\ g\cos\beta \end{pmatrix}$$



EXAMPLE II:
Pressure-driven flow of a Newtonian fluid in a tube:
Poiseuille flow

- steady state
- well developed
- long tube

EXAMPLE III: Pressure-driven flow of a Power-law fluid in a tube



- steady state
- well developed
- long tube

Calculate velocity and stress profiles

Velocity field

Poiseuille flow of a power-law fluid:

$$v_z(r) = \left(\frac{R(L\rho g + P_o - P_L)}{2Lm} \right)^{\frac{1}{n}} \left(\frac{R}{\frac{1}{n}+1} \right) \left(1 - \left(\frac{r}{R} \right)^{\frac{1}{n}+1} \right)$$

Solution to Poiseuille flow in a tube

incompressible, power-law fluid