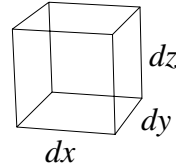


Microscopic Balances

We have been doing microscopic *shell* balances that are specific to whatever problem we are solving.

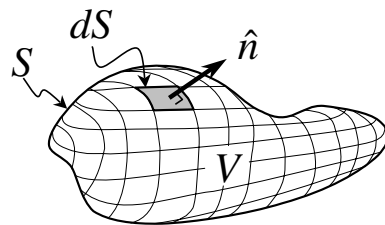


We seek equations for microscopic mass, momentum (and energy) balances that are *general*.

- ⇒
- equations must not depend on the choice of the control volume,
 - equations must capture the appropriate balance

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Continuity Equation



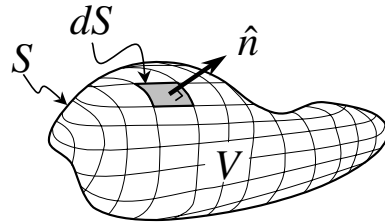
microscopic **mass** balance written on an arbitrarily shaped volume, V , enclosed by a surface, S

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} = -\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

Gibbs notation: $\frac{\partial \rho}{\partial t} + \underline{v} \cdot \nabla \rho = -\rho(\nabla \cdot \underline{v})$

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Equation of Motion



microscopic **momentum** balance written on an arbitrarily shaped volume, V , enclosed by a surface, S

Gibbs notation:
$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla P - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$
 general fluid

Gibbs notation:
$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla P + \mu \nabla^2 \underline{v} + \rho \underline{g}$$
 Newtonian fluid

Navier-Stokes Equation

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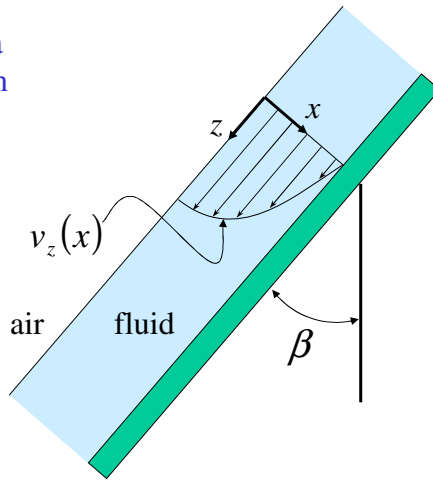
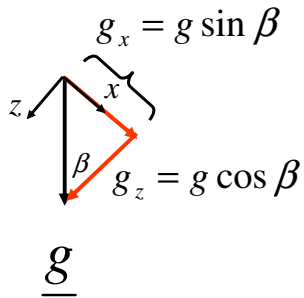
Problem-Solving Procedure - fluid-mechanics problems

1. sketch system
2. choose coordinate system
3. simplify the *continuity* equation (mass balance)
4. simplify the 3 components of the *equation of motion* (momentum balance) (*note that for a Newtonian fluid, the equation of motion is the Navier-Stokes equation*)
5. solve the differential equations for velocity and pressure (if applicable)
6. apply boundary conditions
7. calculate any engineering values of interest (flow rate, average velocity, force on wall)

amended: when using the microscopic balances

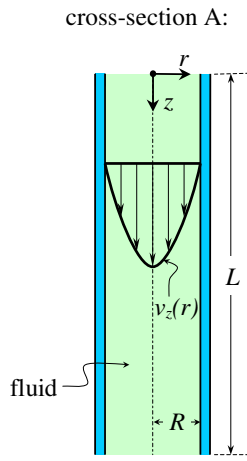
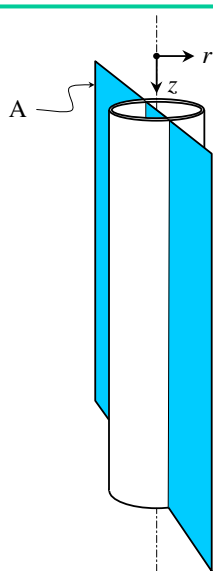
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EXAMPLE I: Flow of a Newtonian fluid down an inclined plane



$$\underline{g} = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} = \begin{pmatrix} g \sin \beta \\ 0 \\ g \cos \beta \end{pmatrix}$$

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EXAMPLE II:

Pressure-driven flow of a Newtonian fluid in a tube:

Poiseuille flow

- steady state
- well developed
- long tube

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EXAMPLE III: Pressure-driven flow of a Power-law fluid in a tube

The diagram shows a 3D view of a tube on the left and a 2D cross-section A on the right. The cross-section A shows a parabolic velocity profile $v_z(r)$ within a tube of radius R and length L . The radial coordinate is r and the axial coordinate is z . The fluid is labeled "fluid". A downward arrow represents gravity \underline{g} .

- steady state
- well developed
- long tube

Calculate velocity and stress profiles

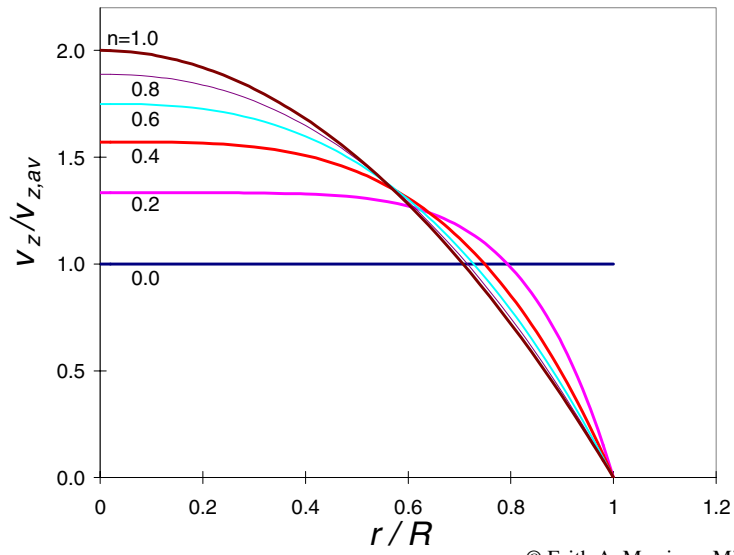
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Velocity field
Poiseuille flow of a power-law fluid:

$$v_z(r) = \left(\frac{R(L\rho g + P_o - P_L)}{2Lm} \right)^{\frac{1}{n}} \left(\frac{R}{\frac{1}{n} + 1} \right) \left(1 - \left(\frac{r}{R} \right)^{\frac{1}{n} + 1} \right)$$

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Solution to Poiseuille flow in a tube
incompressible, power-law fluid



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