

We have discussed using non-dimensionalization to design experiments to be used to **scale-up processes** and equipment.

•If for two systems the scale factors (e.g.  $Re$ ,  $Fr$ ) in the governing equations are the same, the two systems are governed by the same governing equations.

•If the dimensionless boundary conditions are also the same, the two systems are **mathematically identical**

= Dynamic Similarity

We can also use non-dimensionalization to help us to **correlate** experimental results.

What does it mean to correlate?

*If we need to know about the operation of an apparatus and we have the apparatus, then we can learn whatever we need to know about the apparatus by conducting experiments.*

*What if we don't have the apparatus? (we're designing one or comparing the possible performance of one with another)*

**Answer:** we can build a scale model and scale up the experimental results on that;

OR

**Answer:** we can use others' results on scale models and scale up *their* experimental results to *our* needs (no point in re-inventing the wheel)

*Either way, this is called creating a data correlation.*

### Example: Laminar flow in a pipe

For laminar flow of a Newtonian fluid we can calculate the relationship between pressure drop and flow rate. We can also calculate the frictional force on the wall, which is related to these.

(from solution to NS eqns)

$$\tau_{rz} = \frac{r\Delta P}{2L}$$

z-component of force on the wall

$$F_z = \int_0^L \int_0^{2\pi} \tau_{rz} \Big|_{r=R} R d\theta dz = \pi R^2 \Delta P$$

( $v_z(r)$  from solution to NS eqns)

$$Q = \int_0^R \int_0^{2\pi} v_z r dr d\theta = \pi R^2 \langle v_z \rangle$$
$$= \frac{\pi \Delta P R^4}{8\mu L} \quad \text{Hagen-Poiseuille equation}$$

$$F_z = \Delta P \pi R^2 = \left( \frac{8\mu L}{R^2} \right) Q$$

(Tells us what pump we need for a given flow rate, for example)

For laminar flow in any pipe

**What about turbulent flow?**

## Turbulent flow in a pipe

The frictional force on the wall is again related to this expression:

z-component of force on the wall

$$F_z = \int_0^L \int_0^{2\pi} \tau_{rz} \Big|_{r=R} R d\theta dz$$

but without the solution for  $v_z(r)$ ,  
where will we get **this**?

If we cannot solve for  $F_z$ , how will we get  $\Delta P$  as a function of  $Q$  when the flow is turbulent?

**We do not know  $\tau_{rz}(r)$ , but we do know that it comes from the solution to the Navier-Stokes equation and the continuity equation.**

*(we just cannot solve it because turbulent flow is **w**ay too complicated.)*

**What then?**

•We could do experiments.

*(but what if we do not have the system?  
what if it is a design problem?)*

•We (or someone else) could do experiments on a similar system and then we could scale the results.

*ahhh . . . DIMENSIONAL ANALYSIS*

### Dimensionless Force on the Wall

$$F_z = \int_0^L \int_0^{2\pi} \tau_{rz}|_{r=R} R d\theta dz$$
$$= - \int_0^L \int_0^{2\pi} \mu \left( \frac{\partial v_z}{\partial r} \right)_{r=R} R d\theta dz$$

Nondimensionalize:

position:

$$r^* \equiv \frac{r}{D}$$

$$z^* \equiv \frac{z}{D}$$

velocity:

$$v_z^* \equiv \frac{v_z}{V}$$

How shall we nondimensionalize  $F_z$ ?

### Nondimensional Wall Friction

$$f \equiv \frac{F_z}{(\text{area})(\text{kinetic energy})}$$
$$= \frac{F_z}{(2\pi RL) \left( \frac{1}{2} \rho v^2 \right)}$$

Fanning friction factor  
dimensionless wall friction  
in a tube

**Non-dimensional force on the wall:**

$$f = \frac{1}{\pi} \frac{D}{L} \frac{1}{\text{Re}} \int_0^{\frac{L}{D}} \int_0^{2\pi} \left( -\frac{\partial v_z^*}{\partial r^*} \right)_{r^* = \frac{1}{2}} d\theta dz^*$$

$$\Rightarrow f = f\left(\text{Re}, \frac{L}{D}\right)$$

for well developed flow expts show there is no L/D dependence

$$\Rightarrow f = f(\text{Re})$$

**Conclusion:** wall friction,  $f$ , should only correlate (vary) with  $Re$

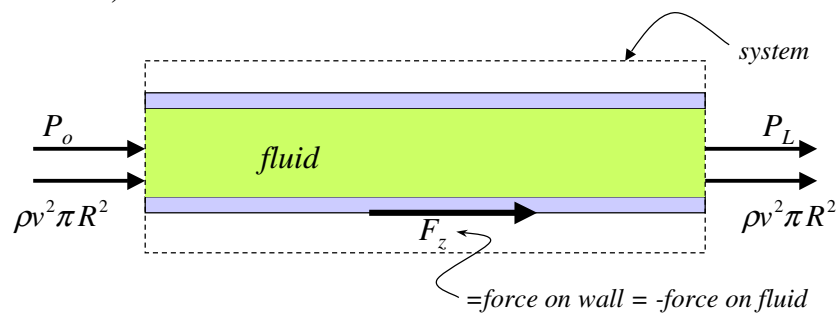
**One final question:**

**How to measure  $f$**

How do we measure  $f$ ?

**Answer:**

We can see how to measure  $f$  by performing a force balance on a straight pipe (incompressible fluid).



Result of force balance (macroscopic momentum balance)  
on straight pipe:

$$F_z = (P_o - P_L)\pi R^2$$

$$f \equiv \frac{F_z}{(\text{area})(\text{kinetic energy})}$$

$$= \frac{F_z}{(2\pi RL)\left(\frac{1}{2}\rho v^2\right)}$$

**Fanning Friction Factor**

$$f = \frac{(P_o - P_L)\pi R^2}{(2\pi RL)\left(\frac{1}{2}\rho v^2\right)} = \frac{(P_o - P_L)\frac{1}{4}}{\left(\frac{L}{D}\right)\left(\frac{1}{2}\rho v^2\right)}$$

Data correlation for friction factor ( $\Delta P$ ) versus Re (flow rate) in a pipe

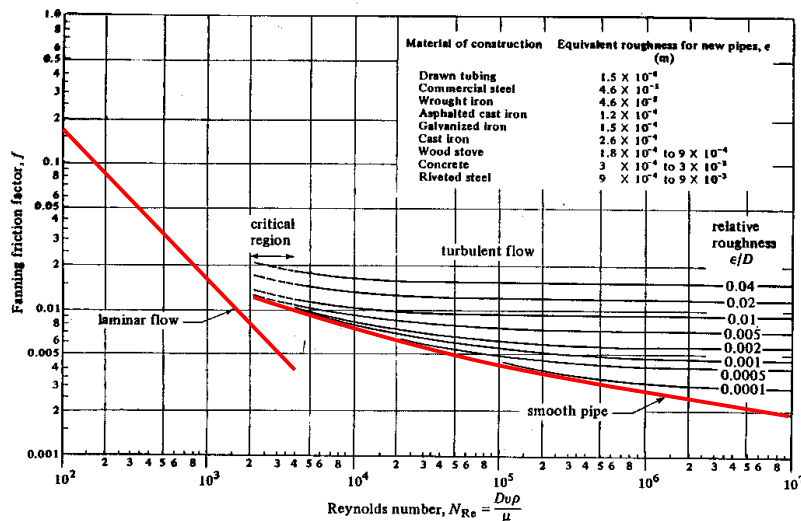


FIGURE 2.10-3. Friction factors for fluids inside pipes. [Based on L. F. Moody, Trans. A.S.M.E., 66, 671, (1944); Mech. Eng. 69, 1005 (1947). With permission.]

(Geankoplis)

## Flow Regimes in a Pipe

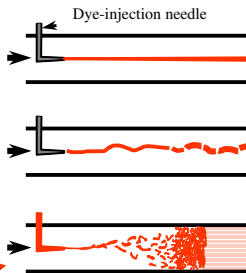
$Re < 2100$  Laminar

- smooth
- one direction only
- predictable

$2100 < Re < 4000$  Transitional

$4000 < Re$  Turbulent

- chaotic - fluctuations within fluid
- transverse motions
- unpredictable - deal with average motion
- most common



## What is the Fanning Friction Factor for Laminar Flow?

$$F_z = \Delta P \pi R^2 = \left( \frac{8\mu L}{R^2} \right) Q$$

$$f = \frac{(P_0 - P_L) \frac{1}{4}}{\left( \frac{L}{D} \right) \left( \frac{1}{2} \rho v^2 \right)} = \frac{16\mu}{\rho v D} = \frac{16}{Re}$$

TRUE!

## Q: What have we done so far?

A: learn to non-dimensionalize

### WHY?

- When flow problems are too complex for analytical or numerical solution, use experimental data correlations. Non-dimensionalization guides the production and use of these data correlations.

## SO FAR . . .

### Microscopic balances

- tell us about flow on microscopic length scale
- yield velocity and stress fields
- require microscopic measurements to verify
- can calculate macroscopic properties
- tell us with assurance what variables influence the flow (either through solving them or through dimensional analysis)
- hard to solve for real systems (complex boundaries, turbulence)**

## NEXT:

### Macroscopic balances

- tell us about flow on macroscopic scale
- yield frictional forces, work, forces on walls, pipes, etc.
- to verify, need to make macroscopic measurements
- easy to solve
- require experimentally determined values for friction (data correlations)**