## Macroscopic Energy Balance

(review from CM2110/CM2120; Felder and Rousseau)

$$\Delta E_k + \Delta E_p + \Delta H = Q_{in} + W_{s,on\ fluid}$$

note: Geankoplis writes $-W_{s,by\ fluid}$

$$\Delta E_k = \sum_{i,out} \frac{1}{2} m_i v_i^2 - \sum_{i,in} \frac{1}{2} m_i v_i^2$$

$$\Delta E_p = \sum_{i,out} m_i g z_i - \sum_{i,in} m_i g z_i$$

$$\Delta H = \sum_{i,out} m_i \hat{H} - \sum_{i,in} m_i \hat{H}$$

$$= \left[ \sum_{i,out} m_i \hat{U} - \sum_{i,in} m_i \hat{U} \right] + \left[ \sum_{i,out} m_i \frac{P_i}{\rho_i} - \sum_{i,in} m_i \frac{P_i}{\rho_i} \right]$$



For the following special case (*which is very common in ChE*), we can simplify the energy balance:

- single-input/single output
- small temperature change
- small $Q_{in}$
- incompressible fluid
- $v$ constant across cross-section

## *Mechanical* Energy Balance

$$\frac{\Delta P}{\rho} + \frac{\Delta\left(v^2\right)}{2} + g\Delta z + \underbrace{\left( \Delta \hat{U} - \frac{Q_{in}}{m} \right)}_{\text{friction, } F} = \frac{W_{s,on}}{m}$$

**Note: $\Delta$ = out-in**

For systems where the magnitude of $\underline{v}$ varies across the cross-section:

$$\langle \Delta E_k \rangle = \sum_{i,out} \left\langle \frac{1}{2} m_i v_i^{\,2} \right\rangle - \sum_{i,in} \left\langle \frac{1}{2} m_i v_i^{\,2} \right\rangle$$

Average across the cross-section for the $i^{th}$ stream

Recall that the average of a function $f$ is calculated from:

$$\langle f(x,y) \rangle = \frac{\iint_A f \, dA}{\iint_A dA} = \frac{1}{A} \iint_A f \, dA$$

$$\langle \Delta E_k \rangle = \frac{1}{A_i} \sum_{i,out} \iint \frac{1}{2} m_i v_i^{\,2} dA_i - \frac{1}{A_i} \sum_{i,in} \iint \frac{1}{2} m_i v_i^{\,2} dA_i$$

$$= \frac{1}{A_i} \sum_{i,out} \iint \frac{1}{2} \rho A_i v_i^{\,3} dA_i - \frac{1}{A_i} \sum_{i,in} \iint \frac{1}{2} \rho A_i v_i^{\,3} dA_i$$

$m_i = \rho A_i v_i$

an average mass/length



$$\langle \Delta E_k \rangle = \frac{1}{A_i} \sum_{i,out} \iint \frac{1}{2} \rho A_i v_i^{\,3} dA_i - \frac{1}{A_i} \sum_{i,in} \iint \frac{1}{2} \rho A_i v_i^{\,3} dA_i$$

Now we substitute $\langle v_i^3 \rangle = \dfrac{1}{A_i} \iint v_i^3 dA_i$ and $m_i = \rho A_i \langle v_i \rangle$

$$\langle \Delta E_k \rangle = \sum_{i,out} \frac{1}{2} m_i \frac{\langle v_i^{\,3} \rangle}{\langle v_i \rangle} - \sum_{i,in} \frac{1}{2} m_i \frac{\langle v_i^{\,3} \rangle}{\langle v_i \rangle}$$

**Following the same logic we used with the macroscopic momentum balance, we define the kinetic energy factor, $\alpha$**

$$\alpha \equiv \frac{\langle v_i \rangle^3}{\langle v_i^3 \rangle}$$

$$\langle \Delta E_k \rangle = \sum_{i,out} \frac{1}{2} m_i \frac{\langle v_i^3 \rangle}{\langle v_i \rangle} - \sum_{i,in} \frac{1}{2} m_i \frac{\langle v_i^3 \rangle}{\langle v_i \rangle}$$

$$= \sum_{i,out} \frac{1}{2\alpha} m_i \langle v_i \rangle^2 - \sum_{i,in} \frac{1}{2\alpha} m_i \langle v_i \rangle^2$$

**experimental result**

$\alpha_{laminar} = 0.5$
$\alpha_{turbulent} = 0.9\text{-}0.99$

$$m_i = \rho A_i \langle v_i \rangle$$



*Mechanical* Energy Balance (with kinetic energy correction)

$$\frac{\Delta P}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\Delta z + F_{friction} = \frac{W_{s,on}}{m}$$

*this is the difference*

**Note: $\Delta$ = out-in**

**Assumptions:**

- single-input/single output
- small temperature change
- small $Q_{in}$
- incompressible fluid
- $\alpha$ accounts for non-plug velocity profile