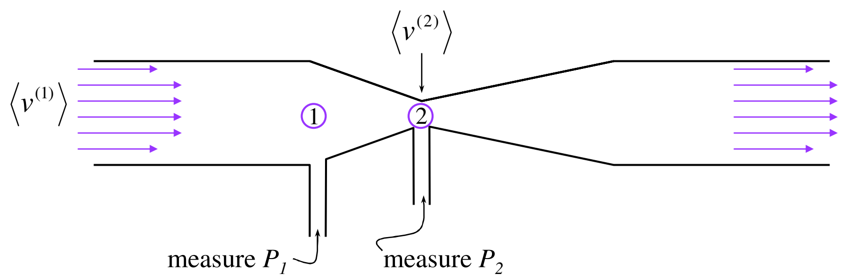
Mechanical Energy Balance Example:
Show how a Venturi meter can be used to measure flow rate in a pipe.



How do we get $F_{friction}$?

Geankoplis 2.10f, p92

Answer: we use data correlations based on dimensional analysis.

Friction in straight pipe, Newtonian fluid
Governing equation:

$$F_z = \int_0^L \int_0^{2\pi} \tau_{rz}\Big|_{r=R} R\, d\theta\, dz = -\int_0^L \int_0^{2\pi} \mu \left(\frac{\partial v_z}{\partial r}\right)\Bigg|_{r=R} R\, d\theta\, dz$$

We're **NOT** assuming laminar flow or well developed flow

Dimensional Analysis

$$f \equiv \frac{F_z}{(area)(kinetic\ energy)} = \frac{F_z}{(2\pi RL)\left(\frac{1}{2}\rho v^2\right)}$$

Fanning friction factor
dimensionless wall friction
in a tube

to determine what f is a function of, we <u>non-dimensionalize</u> the governing equations

Non-dimensional force on the wall:

$$f = \frac{1}{\pi}\frac{D}{L}\frac{1}{\mathrm{Re}}\int_0^{\frac{L}{D}}\int_0^{2\pi}\left(-\frac{\partial v_z^*}{\partial r^*}\right)\bigg|_{r^*=\frac{1}{2}} d\theta\, dz^*$$

$$\Rightarrow f = f\left(\mathrm{Re}, \frac{L}{D}\right)$$

for well developed flow expts show there is no L/D dependence

$$\Rightarrow f = f(\mathrm{Re})$$

Conclusion: wall friction should only correlate with *Re*

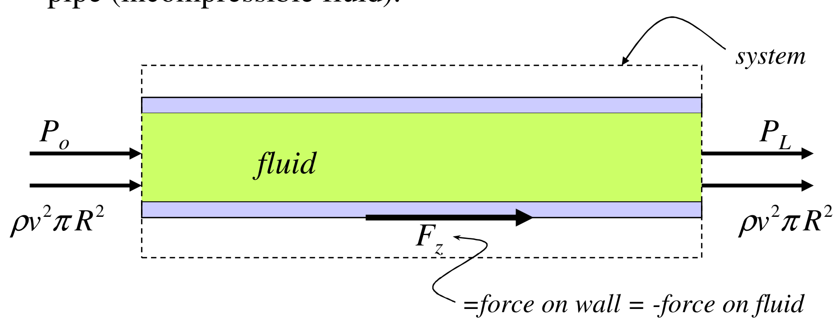
How to measure f

One final question:

How do we measure f ?

Answer:

We can see how to measure f by performing a macroscopic momentum balance on a straight pipe (incompressible fluid).



Result of momentum balance on straight pipe:

$$F_z = (P_o - P_L)\pi R^2$$

$$f \equiv \frac{F_z}{(area)(kinetic\ energy)}$$

$$= \frac{F_z}{(2\pi RL)\left(\frac{1}{2}\rho v^2\right)}$$

Fanning Friction Factor

$$f = \frac{(P_o - P_L)\pi R^2}{(2\pi RL)\left(\frac{1}{2}\rho v^2\right)} = \frac{(P_o - P_L)\frac{1}{4}}{\left(\frac{L}{D}\right)\left(\frac{1}{2}\rho v^2\right)}$$

How do we use f in the Mechanical Energy Balance (MEB)?

MEB:

$$\frac{\Delta P}{\rho} + \frac{\Delta(v^2)}{2\alpha} + g\Delta z + F_{friction} = \frac{W_{s,on}}{m}$$

we can rearrange f to look like this term

use in MEB

$$4f\left(\frac{L}{D}\right)\left(\frac{1}{2}v^2\right) = \left(\frac{(\Delta P)}{\rho}\right)$$

wall friction in straight pipes in units that match the MEB

friction in straight pipes

GETTING THE FRICTION TERM

Procedure:
1. calculate *Re*
2. look up f
3. calculate $F_{friction}$
4. use in MEB

straight pipes:

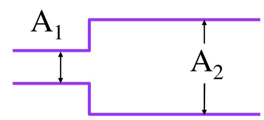
$$F_{friction} = 4f\left(\frac{L}{D}\right)\left(\frac{1}{2}v^2\right)$$

What about bends, valves, fittings, etc.?

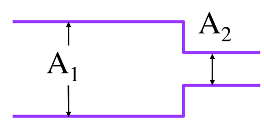
Answer: same drill: perform MEB, obtain terms that vary with velocity and that contain geometric variables

Friction in fittings, bends, etc.

$$F_{\text{expansion}} = \frac{1}{\alpha}\left(1-\frac{A_1}{A_2}\right)^2\left(\frac{\left(v^{(1)}\right)^2}{2}\right)$$



$$F_{\text{contraction}} = \frac{0.55}{\alpha}\left(1-\frac{A_2}{A_1}\right)\left(\frac{\left(v^{(2)}\right)^2}{2\alpha}\right)$$



$$F_{\text{fittings}} = K\left(\frac{v^2}{2}\right)$$

see table 2.11-1, p93 Geankoplis for values of K

Friction term in Mechanical Energy Balance

length of straight pipe

number of each type of fitting

$$F_{friction} = \left(4f\frac{L}{D}+\sum_i K_i n_i\right)\frac{v^2}{2}$$

friction-loss coefficients