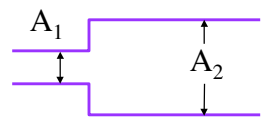
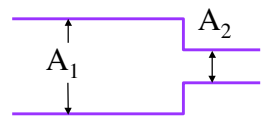
Friction in fittings, bends, etc.

$$F_{\text{expansion}} = \frac{1}{\alpha}\left(1-\frac{A_1}{A_2}\right)^2\left(\frac{\left(v^{(1)}\right)^2}{2}\right)$$



$$F_{\text{contraction}} = \frac{0.55}{\alpha}\left(1-\frac{A_2}{A_1}\right)\left(\frac{\left(v^{(2)}\right)^2}{2\alpha}\right)$$



$$\boxed{F_{\text{fittings}} = K\left(\frac{v^2}{2}\right)}$$

see table 2.11-1, p93 Geankoplis for values of K

Friction term in Mechanical Energy Balance

length of straight pipe

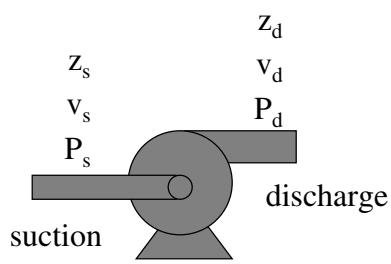
number of each type of fitting

$$F_{friction} = \left(4f\frac{L}{D} + \sum_i K_i n_i\right)\frac{v^2}{2}$$

friction-loss coefficients

Centrifugal Pumps

$$\frac{\Delta P}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z + F_{friction} = \frac{W_{s,on}}{m}$$



Centrifugal Pumps - MEB written on suction/discharge system

$$\frac{\Delta P}{\rho} + \frac{\Delta\left(v^2\right)}{2\alpha} + g\Delta z = \underbrace{\frac{W_{s,on}}{m} - F_{friction}}$$

pump efficiency

$$\boxed{\frac{\eta W_{s,by\ pump}}{m}}$$

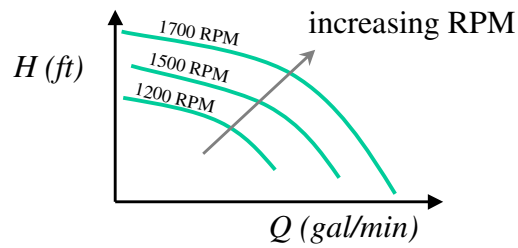
electric motor efficiency

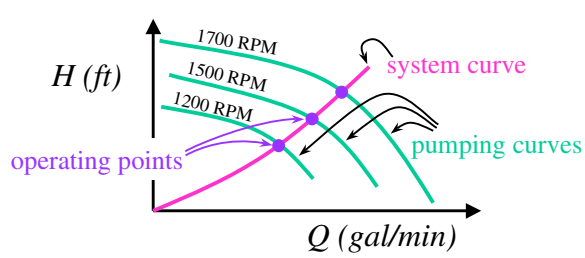
brake horsepower

$$\boxed{W_{s,by\ pump} = \eta_e (current)(voltage)}$$

Pumping Head - pump characteristic curve

$$\underbrace{\overbrace{\frac{\Delta P}{g\rho}}^{\text{pressure head}} + \overbrace{\frac{\Delta\left(v^2\right)}{2g\alpha}}^{\text{velocity head}} + \overbrace{\Delta z}^{\text{elevation head}}}_{\text{written around pump}} = \frac{\eta W_{s,by\ pump}}{mg} = \underbrace{H}_{\textbf{Total head}} = \left(\frac{\text{energy}}{\text{weight}}\right)$$



Pumping Head - system curve

$$\underbrace{\overbrace{\frac{\Delta P}{g\rho}}^{\text{pressure head}} + \overbrace{\frac{\Delta\left(v^2\right)}{2g\alpha}}^{\text{velocity head}} + \overbrace{\Delta z}^{\text{elevation head}}}_{\text{written around system}} + \frac{F_{system}}{g} = \underbrace{H}_{\textbf{Total head required by system}} = \left(\frac{\text{energy}}{\text{weight}}\right)$$