

Compressible Fluids

- most fluids are somewhat compressible
- in chemical-engineering processes, compressibility is unimportant at most operating pressures
- even gases may be modeled as incompressible if $\Delta p < p_{\text{mean}}$

EXCEPT:

When the fluid velocity approaches the speed of sound

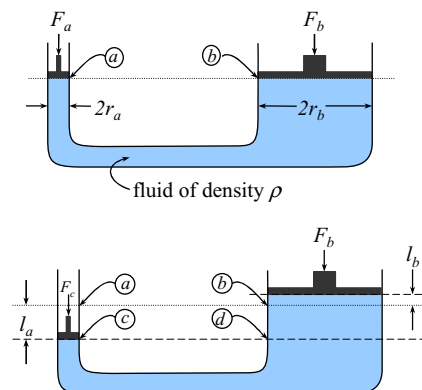
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Compressible Fluids

How is pressure information transmitted in liquids and gases?

The Hydraulic Lift operates on Pascal's principle

Pressure exerted on an enclosed liquid is transmitted equally to every part of the liquid and to the walls of the container.



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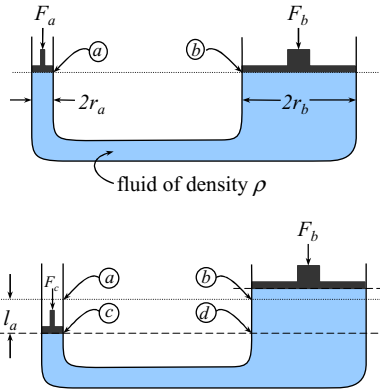
Compressible Fluids

For static incompressible liquids,

The Hydraulic Lift operates on Pascal's principle

Pressure exerted on an enclosed liquid is transmitted equally to every part of the liquid and to the walls of the container.

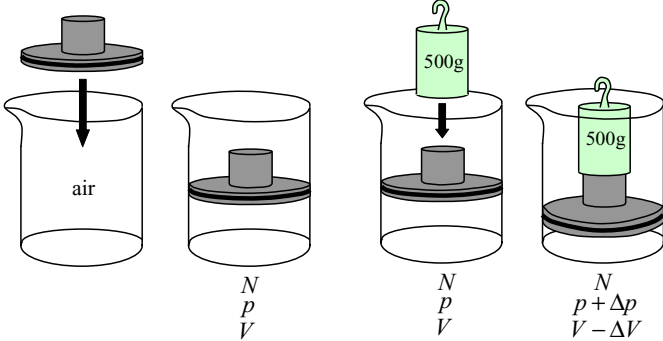
and essentially, instantaneously



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Compressible Fluids

For static compressible fluids (gases), pressure causes volume change.

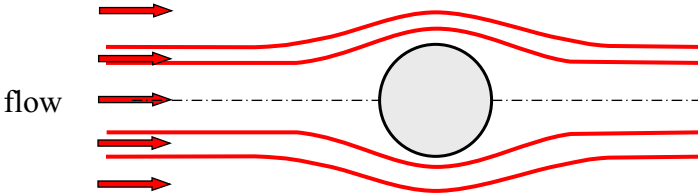


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Compressible Fluids

For moving incompressible liquids and gases,

The presence of the obstacle is felt by the upstream fluid (pressure) and that information is transmitted very rapidly throughout the fluid.



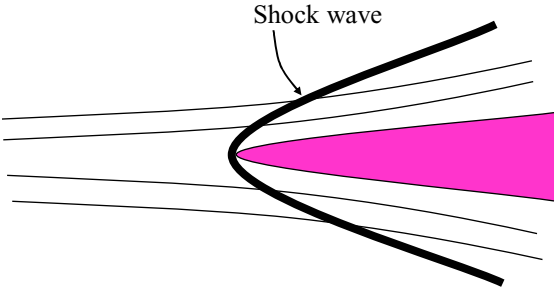
The streamlines adjust according to momentum conservation.

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Compressible Fluids

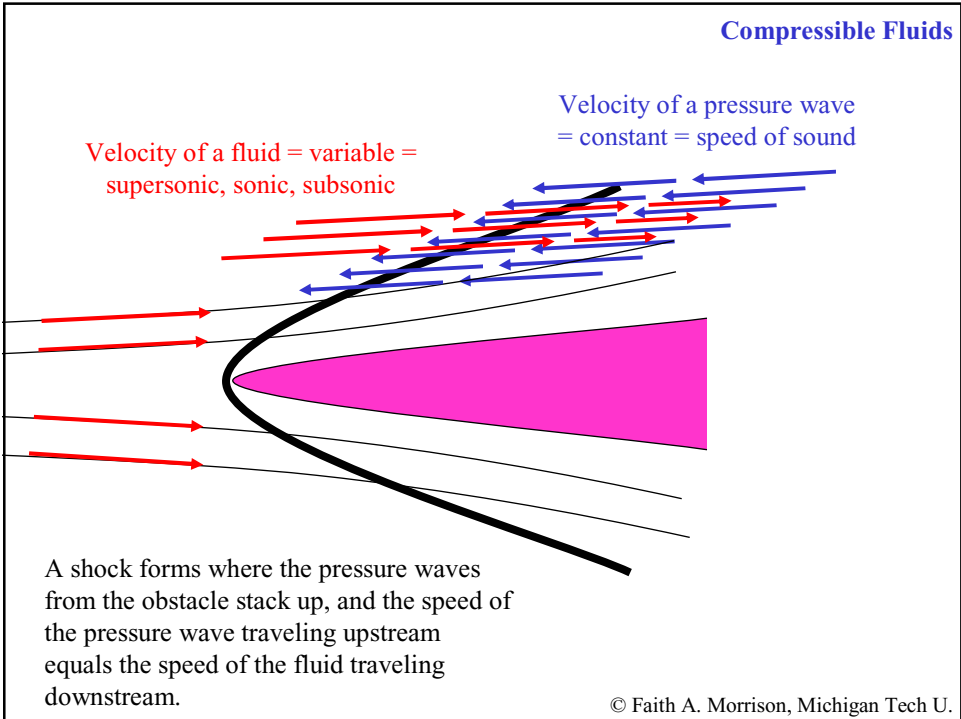
For compressible fluids moving near sonic speeds, information (pressure) and the gas itself are moving at comparable speeds.

Pressure piles up at the shock wave



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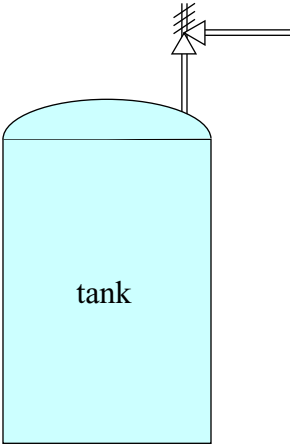
Compressible Fluids



Compressible Fluids

Relief Valves (Safety Valves)

The rapid flows in relief valves can become sonic.



For sonic flow, the flow rate is constant no matter what the pressure drop is.

(pressure waves pile up)

Choked Flow

Choked flow can be understood from basic equations of compressible fluid mechanics.

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Momentum and Energy in Compressible Fluids

Microscopic momentum balance:

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

incompressible

$$\underline{\underline{\tau}} = -\mu (\nabla \underline{v} + (\nabla \underline{v})^T)$$

compressible

$$\underline{\underline{\tau}} = -\mu (\nabla \underline{v} + (\nabla \underline{v})^T) + \left(\frac{2}{3} \mu - \kappa \right) \nabla \cdot \underline{v}$$

κ = bulk viscosity

Mechanical energy balance:

incompressible

$$\frac{\Delta p}{\rho} + \frac{\Delta V^2}{2\alpha} + g\Delta z + F = \frac{W_{s,on}}{\dot{m}}$$

compressible?

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Mechanical energy balance (compressible)

Back up one step and reintegrate without constant ρ assumption.

$$\frac{dp}{\rho} + VdV + gdz + dF = \frac{dW_{s,on}}{\dot{m}}$$

Assume:

- constant cross section
- constant mass flow $\rho AV = GA$
- neglect gravity
- no shaft work

$$G \equiv \rho V = \text{mass velocity}$$

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Mechanical energy balance (compressible)

Ideal Gas Law $pV = NRT$

$$\frac{V}{N} = \frac{RT}{p}$$

$$\frac{V}{MN} = \frac{RT}{pM}$$

$$\frac{1}{\rho} = \frac{RT}{pM}$$

For isothermal flow:

$$p_1 V_1 = NRT$$

$$p_2 V_2 = NRT$$

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Also, $\frac{\rho_{av}}{p_{av}} = \frac{M}{RT}$

$$\frac{2\rho_{av}}{p_1 + p_2} = \frac{M}{RT}$$

Mechanical energy balance
(compressible) $G \equiv \rho V = \text{mass velocity}$

$$(p_2 - p_1) + \frac{G^2}{\rho_{av}} \ln \frac{p_1}{p_2} + \frac{2fG^2}{\rho_{av}D} (L_2 - L_1) = 0$$

(matches equation 3.187 in handout)

The compressible MEB predicts that there is a maximum velocity at

(see handout)

$$V_{\max} = \sqrt{\frac{p_2}{\rho_2}} = \sqrt{\frac{RT}{M}} = \text{isothermal speed of sound}$$

A better assumption than isothermal flow is adiabatic flow (no heat transferred). For this case,

$$V_{\max} = \sqrt{\frac{\gamma p_2}{\rho_2}} = \sqrt{\frac{\gamma RT}{M}} = \text{adiabatic speed of sound}$$
$$\gamma = \frac{C_p}{C_v}$$

(see handout)