Applied Heat Xfer

21 Nov 19

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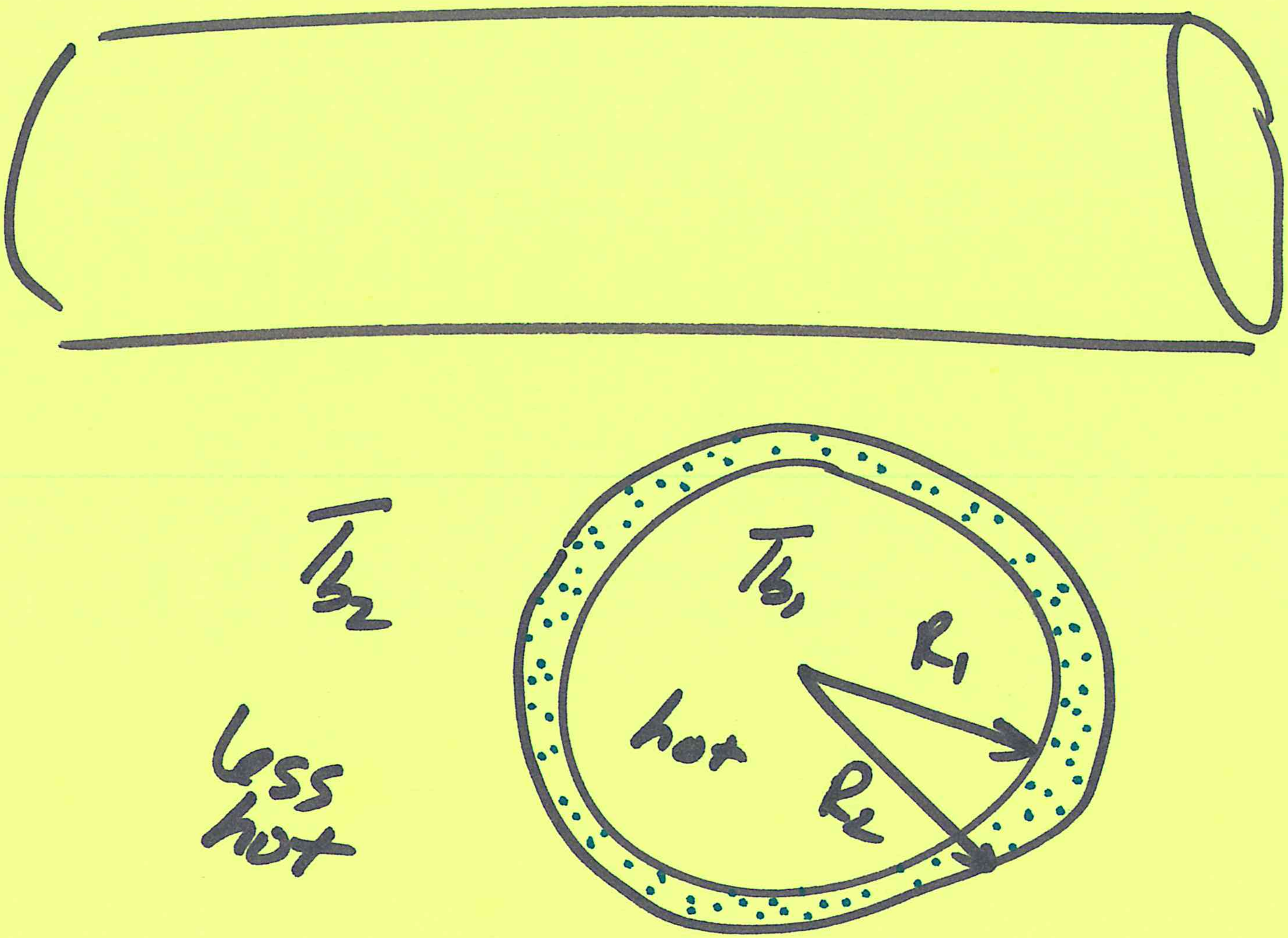
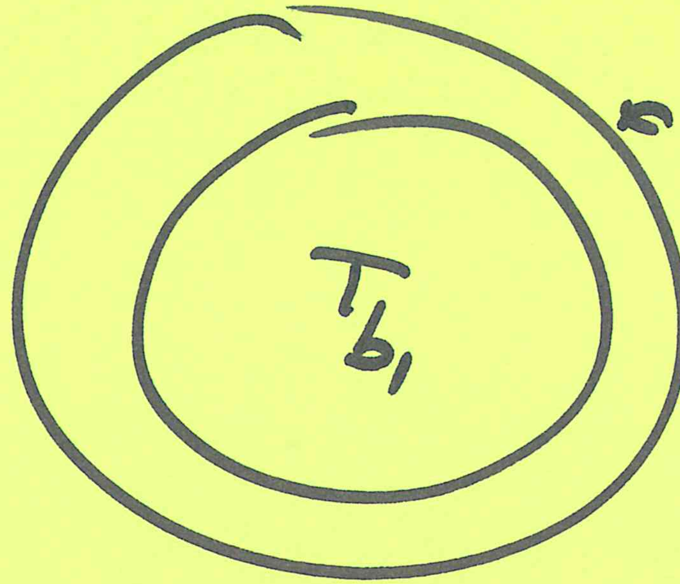
So far:





$$\left.\frac{q_r}{A}\right|_{R_2} = h(T_{w_2} - T_{b_2})$$

T_{b_2}

$T_{b_1} > T_{b_2}$

$(T_{b_1} - T_{b_2})$ = driving force

This is what causes heat xfr

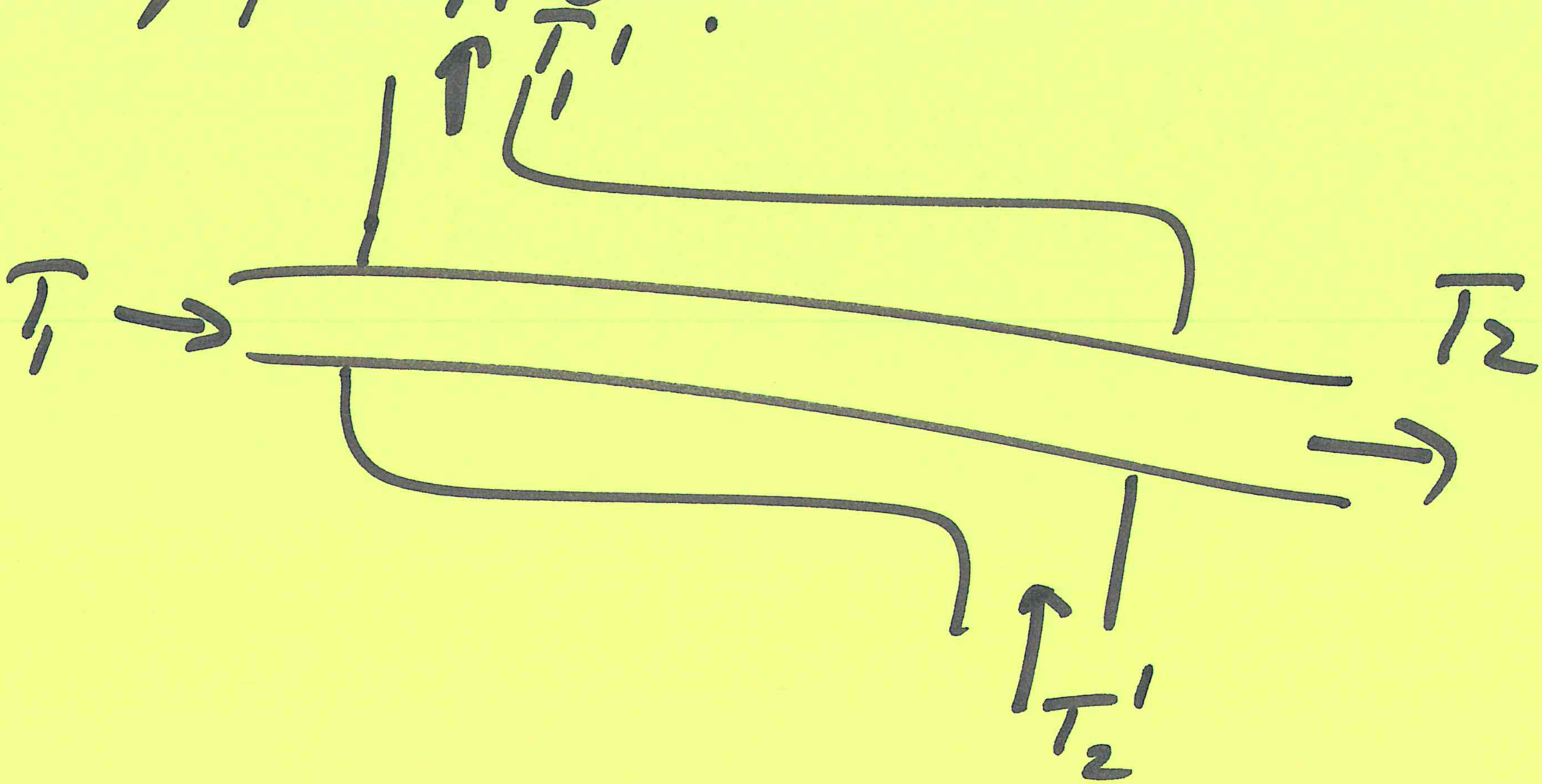
How do we characterize the amount of heat transfer in the slice, overall?

$$Q = \frac{q_r}{A}(2\pi r L) = \left(\frac{\frac{1}{R_1}}{\frac{1}{h_1 R_1} + \frac{1}{k}\ln\frac{R_2}{R_1} + \frac{1}{h_2 R_2}}\right) 2\pi L R_1 \left(T_{b_1} - T_{b_2}\right)$$

$$= \underbrace{\left(U\right)}_{\text{new defined qty}} \left(\text{AREA}\right)\left(\text{DRIVING FORCE}\right)$$

OUR QUESTION:

What is the correct average driving force for heat transfer in a double pipe HE?



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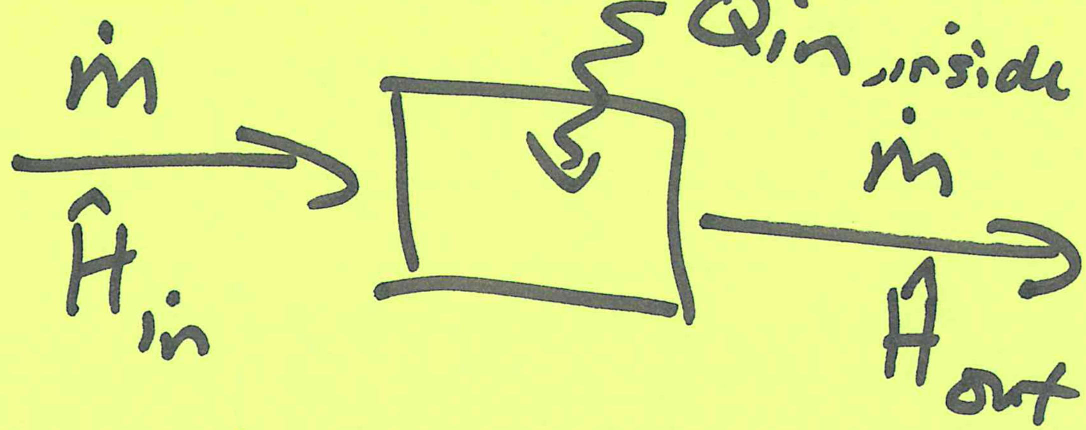
MACRO E-BAL - INSIDE



no stirrer turbine pump

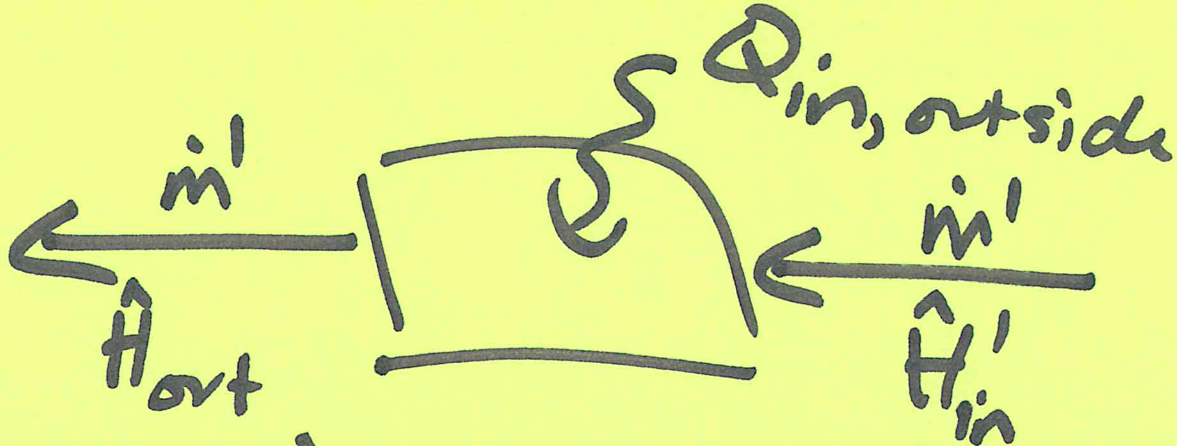
$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = Q_{in} + \cancel{W_{s,in}}$$

$$Q_{in} = \Delta H$$

$$= \sum_{outs} \dot{m}_i \hat{H}_i - \sum_{in} \dot{m}_i \hat{H}_i$$

$$= \dot{m} (\hat{H}_{out} - \hat{H}_{in})$$

MACRO E-BAL OUTSIDE



$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = \dot{Q}_{in} + \cancel{W_s}$$

$$\dot{Q}_{in} = \Delta H$$

$$= \sum_{outs} \dot{m}_i' \hat{H}_i' - \sum_{in} \dot{m}_i' \hat{H}_i'$$

$$= \dot{m}' (\hat{H}'_{out} - \hat{H}'_{in})$$

$$= \dot{m}' C_p (T'_{out} - T'_{in})$$

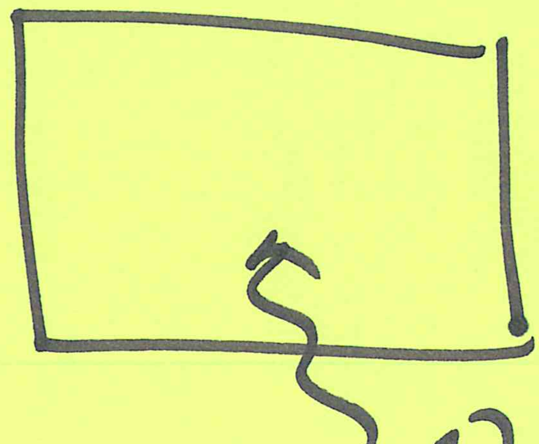
$$\dot{Q}_{in} = \dot{m}\int_{T_{in}}^{T_{out}} C_p \, dT$$

$$\boxed{\dot{Q}_{in} = \dot{m} C_p (T_{out} - T_{in})}$$

Now,
overall →

$$Q_{in,outside} = \dot{m}' C_p' (T_1' - T_2')$$

MACRO E-BAL OVERALL



$Q_{in} = 0$

Adiabatic

$$\cancel{\Delta E_p} + \cancel{\Delta E_k} + \Delta H = \cancel{Q_{in}} + \cancel{W_{s,in}}$$

$$\Delta H = 0 = Q_{in,inside} = Q_{in,outside}$$

After subtracting the MACRO E-BAL RESULTS:

$$\underbrace{\frac{dQ_{in}}{dx}}_{U\,2\pi R(T'-T)}\left(\frac{1}{\dot{m}c_p'}-\frac{1}{\dot{m}c_p}\right)=\frac{d(\overbrace{T'-T}^{\equiv\Phi})}{dx}$$

$$\frac{d\Phi}{dx}=\underbrace{\left(U\,2\pi R\left(\frac{1}{\dot{m}c_p'}-\frac{1}{\dot{m}c_p}\right)\right)}_{\alpha_0}\Phi$$

$$\frac{d\Phi}{\Phi} = \alpha_0 \, dx$$

$$\ln \Phi = \alpha_0 x + C_1$$

use this to get C_1

BC: $\boxed{x = 0 \qquad \Phi = T_1' - T_1}$

$x = L \qquad \Phi = T_2' - T_2$

Solve for $T' - T$ as a function of x //

But what is correct average driving force for heat transfer in a double pipe HE?

$$\ln\left(\frac{T_2' - T_2}{T_1' - T_1}\right) = U\,(2\pi R L)\underbrace{\left(\frac{1}{c_p' m'} - \frac{1}{c_p m}\right)}$$

write in terms of Q

USING MACRO EBAL

. . . (algebra) (inside, outside)

Result

$$Q = U \overbrace{(2\pi R L)}^{A} \Delta T_{lm}$$

$$\boxed{\Delta T_{lm} = \frac{\Delta T_{left} - \Delta T_{right}}{\ln \dfrac{\Delta T_{left}}{\Delta T_{right}}}}$$