Part I: Momentum Transfer

Momentum transfer:

$$\underbrace{\tau_{21}}_{\text{momentum flux}} = -\mu \underbrace{\frac{dv_1}{dx_2}}_{\text{velocity gradient}}$$

Part II: Heat Transfer

Heat transfer:

$$\underbrace{\frac{q_x}{A}}_{\text{heat flux}} = -k \underbrace{\frac{dT}{dx}}_{\text{temperature gradient}}$$

k: thermal conductivity

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Heat Transfer

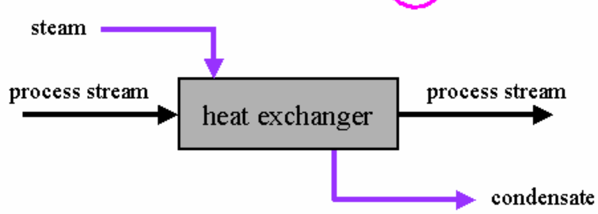
Need amount of heat transferred to system

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on}$$

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on}$$



Fourier's law of Heat Conduction:

makes reference to a coordinate system

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

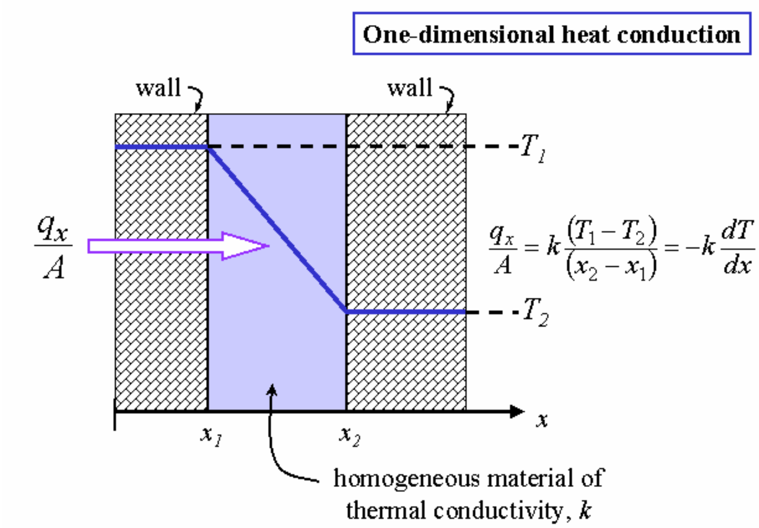
Allows you to solve for temperature profiles

$$\frac{\underline{q}}{A} = -k\nabla T \quad \text{Gibbs notation}$$

•Heat flows down a temperature gradient

•Flux is proportional to temperature gradient

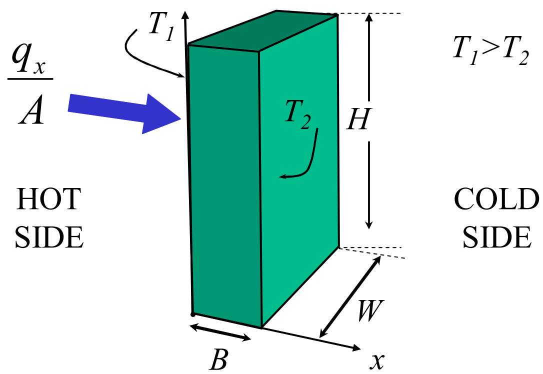
Fourier's Experiments: Simple One-dimensional Heat Conduction



Example 1: Heat flux in a rectangular solid

Assumptions:
- wide, tall slab
- steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?



Energy Balance

$$\begin{pmatrix} rate\ of \\ energy \\ accumulation \end{pmatrix} = \begin{pmatrix} rate\ of \\ energy \\ in \end{pmatrix} - \begin{pmatrix} rate\ of \\ energy \\ out \end{pmatrix} + \begin{pmatrix} rate\ of \\ energy \\ production \end{pmatrix}$$

Convective and *conduction* terms - energy that passes through boundaries

e.g. chemical reaction, electrical current

conduction - Fourier's law

convection - due to flow

Problem-Solving Procedure - heat-transfer problems

1. sketch system
2. choose coordinate system
3. choose a control volume - small dimension in the direction of flux
4. perform an energy balance (will contain energy flux)
5. substitute in *Fourier's law of conduction*, e.g. $\frac{q_x}{A} = -k\left(\frac{dT}{dx}\right)$
6. solve the <u>differential</u> equation for temperature profile
7. apply boundary conditions

does this seem familiar?

Example 1: Heat flux in a slab

Solution:

$$\frac{q_x}{A} = c_1$$

Constant

$$T = \frac{-c_1}{k}x + c_2$$

Boundary conditions?

Example 1: Heat flux in a slab; temperature boundary conditions

Solution:

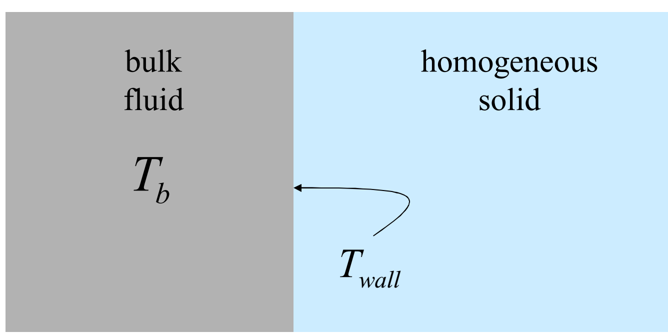
$$\frac{q_x}{A} = -k\left(\frac{T_2 - T_1}{B}\right)$$

Constant, and depends on k

$$T = \left(\frac{T_2 - T_1}{B}\right)x + T_1$$

Varies linearly, and **<u>does not</u>** depend on k

An Important Boundary Condition in Heat Transfer: <u>Newton's Law of Cooling</u>



$$T_b \neq T_{wall}$$

What is the flux at the wall?

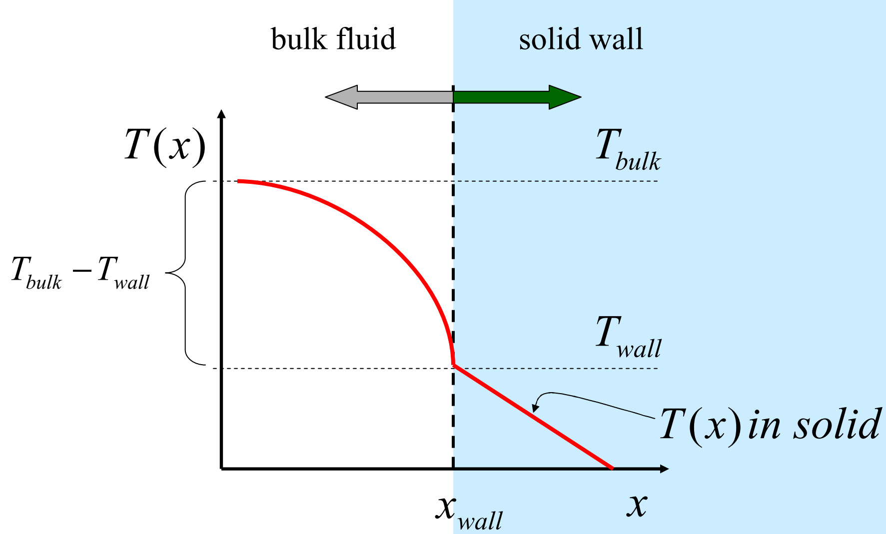
The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

h depends on:

- geometry
- fluid velocity
- fluid properties
- temperature difference



The temperature difference at the fluid-wall interface is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

How do we handle the absolute value signs?

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

•Heat flows from hot to cold

•The coordinate system determines if the flux is positive or negative

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