

Analysis of double-pipe heat exchanger (cont)

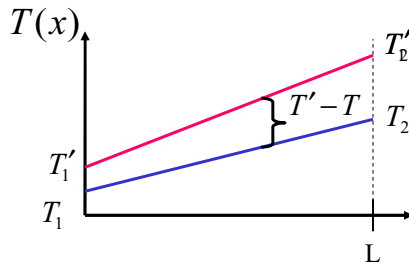
Question: How can we write $\frac{dQ_{in}}{dx}$ in terms of $T'-T$?

Answer: Use overall heat transfer coefficient, U

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Result:
$$\frac{d(T' - T)}{dx} = \frac{dQ_{in}}{dx} \left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m} \right)$$

want to solve for $T' - T$,
but this is a function of $T' - T$



We can write dQ_{in}/dx in terms of $T' - T$ if we introduce the overall heat transfer coefficient, U

$$\begin{aligned} dQ_{in} &= UA(T' - T) \\ &= U(2\pi R dx)(T' - T) \end{aligned}$$

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Temperature profile in a double-pipe heat exchanger:

$$\frac{T' - T}{T'_1 - T_1} = e^{\alpha_0 x} \quad \alpha_0 = 2\pi R U \left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m} \right)$$

Useful result, but what we REALLY want is an easy way to relate $Q_{in,overall}$ to inlet and outlet temperatures.

Begin by evaluating temperature profile at exit conditions:

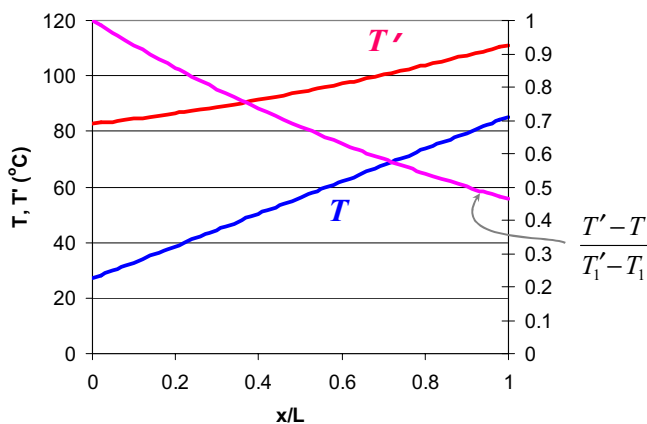
$$\ln \left(\frac{T'_2 - T_2}{T'_1 - T_1} \right) = U(2\pi RL) \left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m} \right)$$

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Note that the temperature curves are not necessarily linear.

$$\frac{T' - T}{T'_1 - T_1} = e^{\alpha_0 L \left(\frac{x}{L} \right)}$$

$$\alpha_0 L = 2\pi RL U \left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m} \right)$$



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$$\ln\left(\frac{T'_2 - T_2}{T'_1 - T_1}\right) = U(2\pi RL) \left(\frac{1}{\hat{C}'_p m'} - \frac{1}{\hat{C}_p m} \right)$$

The $m\hat{C}_p$ terms appear in the overall macroscopic energy balances. We can then rearrange this equation to look like this:

$$Q_{in,overall} = m\hat{C}_p \Delta T$$

$$= -m'\hat{C}'_p \Delta T'$$

$$Q_{in,overall} = UA \Delta T_{average}$$

total heat transferred in exchanger

average temperature driving force

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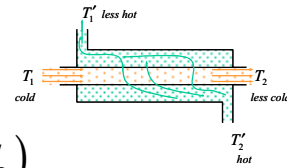
FINAL RESULT:

$$Q = U \underbrace{(2\pi RL)}_A \underbrace{\frac{(T'_1 - T_1) - (T'_2 - T_2)}{\ln \frac{(T'_1 - T_1)}{(T'_2 - T_2)}}}_{\Delta T_{lm}}$$

$$Q = UA \Delta T_{lm}$$

$\equiv \Delta T_{lm}$
=log-mean temperature difference

ΔT_{lm} is the correct average temperature to use for the overall heat-transfer coefficients in a double-pipe heat exchanger.



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Example: Heat Transfer in a Double-Pipe Heat Exchanger: *Geankoplis 4th ed. 4.5-4*

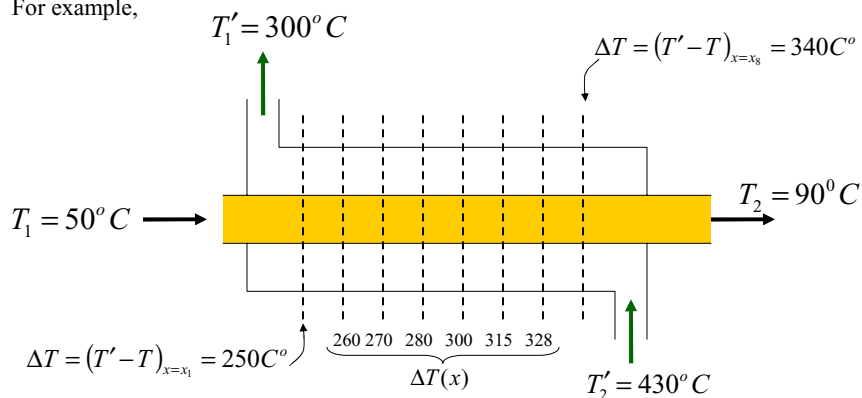
Water flowing at a rate of 13.85 kg/s is to be heated from 54.5 to 87.8°C in a double-pipe heat exchanger by 54,430 kg/h of hot gas flowing counterflow and entering at 427°C ($C_{pm}=1.005$ kJ/kg K). The overall heat-transfer coefficient based on the outer surface is $U_o = 69.1$ W/m² K. Calculate the exit-gas temperature and the heat transfer area needed

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Summary:

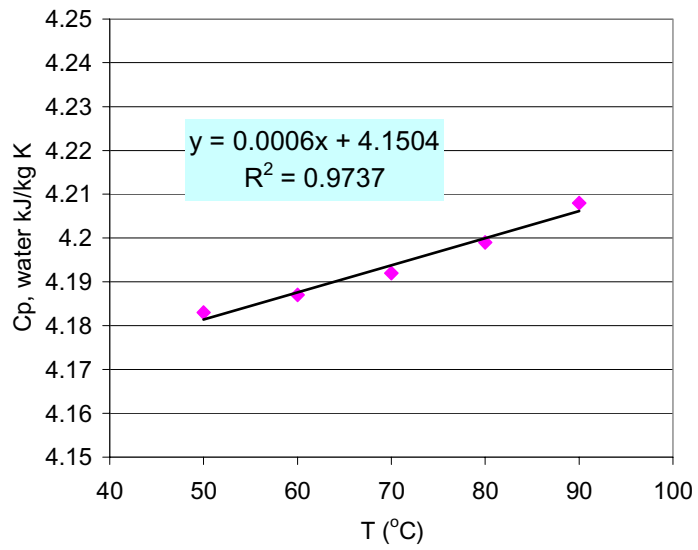
Double-Pipe Heat Exchanger – the driving force for heat transfer changes along the length of the heat exchanger

For example,



The correct *average* driving force for the whole exchanger is the log-mean temperature difference $= \Delta T_{lm}$

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