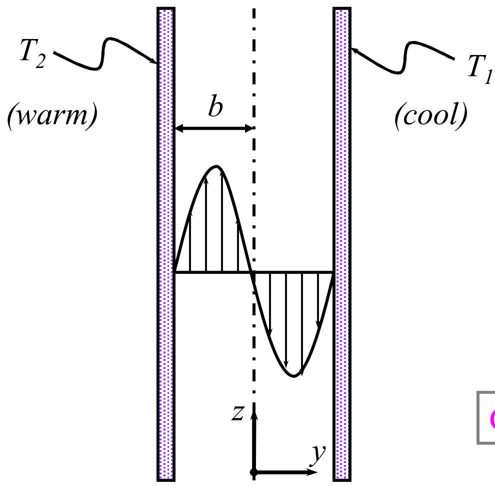
Example: Free convection between long parallel plates *or* heat transfer through double-pane glass windows



$$T_2 > T_1$$

assumptions:
long, wide slit
steady state
no source terms
viscosity constant
density varies with T

Calculate: T, $\underline{v}$ profiles

In our analyses of momentum transport so far, we have assumed constant density

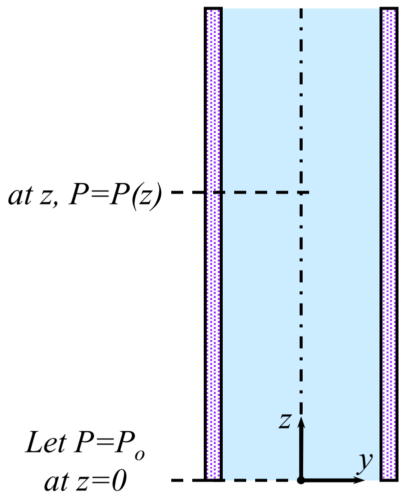
$\Longrightarrow$ use Navier-Stokes equation

Actually, we can use the Navier-Stokes equation for any problem for which the following equation holds:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Is Pressure a function of z?

YES, there should be hydrostatic pressure (ρgh)



"Pressure at the bottom of a column of fluid = pressure at top + ρgh."

average density

$$P_o = P(z) + \bar{\rho} g z$$

$$\Rightarrow P(z) = P_o - \bar{\rho} g z$$

$$\Rightarrow \frac{dP}{dz} = -\bar{\rho} g$$

To account for the temperature variation of ρ:

$$\rho = \bar{\rho} - \bar{\rho}\bar{\beta}\left(T - \bar{T}\right)$$

$\bar{\rho} =$ mean density

$\bar{\beta} =$ volumetric coefficient of expansion at $\bar{T}$

$$\bar{T} = \frac{T_1 + T_2}{2}$$

Final Result: (free convection between two slabs)

$$v_z(y) = \frac{\bar{\rho}\bar{\beta}g(T_2 - T_1)b^2}{12\mu}\left[\left(\frac{y}{b}\right)^3 - \left(\frac{y}{b}\right)\right]$$

(see next slide for plot)