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z-component of the Navier-Stokes Equation:

$$\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

**Choose:**

**D = characteristic length**  
**V = characteristic velocity**  
**D/V = characteristic time**  
 **$\rho V^2$  = characteristic pressure**

**This velocity is an imposed (forced) average velocity**

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non-dimensional variables:

time: $t^* \equiv \frac{tV}{D}$	position: $r^* \equiv \frac{r}{D}$ $z^* \equiv \frac{z}{D}$	velocity: $v_z^* \equiv \frac{v_z}{V}$ $v_r^* \equiv \frac{v_r}{V}$ $v_\theta^* \equiv \frac{v_\theta}{V}$	driving force: $P^* \equiv \frac{P}{\rho V^2}$ $g_z^* \equiv \frac{g_z}{g}$
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z-component of the  
nondimensional Navier-Stokes  
Equation:

$$\frac{Dv_z^*}{Dt} = -\frac{\partial P^*}{\partial z^*} + \frac{\mu}{\rho V D} (\nabla^2 v_z)^* + \frac{gD}{V^2} g^*$$

$\frac{1}{Re}$       $\frac{1}{Fr}$

$$(\nabla^2 v_z)^* \equiv \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial v_z^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left( \frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*} \frac{\partial v_z^*}{\partial \theta} + v_z^* \frac{\partial v_z^*}{\partial z^*} \right)$$

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*For more complex systems involving free convection,  
what are the appropriate dimensionless groups with  
which to correlate data?*

for forced convection we used:  $v_z^* = \frac{v_z}{V}$

for free convection  $\bar{v} = 0$ ; what V should we use?

use a Reynolds-number type expression:

$$v_z^* = \frac{\bar{\rho} v_z D}{\mu} \Rightarrow V = \frac{\mu}{D \bar{\rho}}$$

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When non-dimensionalizing the Navier-Stokes, what do I use for  $\rho$ ?

$$\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right) + \rho g_z$$

here we use  $\bar{\rho}$  because the issue is volumetric flow rate

as before, for pressure gradient we use  $-\bar{\rho}g$

here we use  $\rho(T)$  because the issue is driving the flow by density differences affected by gravity

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non-dimensional variables:

<p>time:</p> $t^* \equiv \frac{t\mu}{D^2 \bar{\rho}}$	<p>position:</p> $r^* \equiv \frac{r}{D}$ $z^* \equiv \frac{z}{D}$	<p>velocity:</p> $v_z^* \equiv \frac{v_z D \bar{\rho}}{\mu}$ $v_r^* \equiv \frac{\bar{\rho} v_r D}{\mu}$ $v_\theta^* \equiv \frac{\bar{\rho} v_\theta D}{\mu}$	<p>driving force:</p> $T^* = \frac{T - \bar{T}}{T_2 - \bar{T}}$
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Example: how do we non-dimensionalize  $\frac{\partial v_z}{\partial t}$  ?

$$v_z^* \equiv \frac{v_z D \bar{\rho}}{\mu} \Rightarrow v_z = \frac{v_z^* \mu}{D \bar{\rho}}$$

$$t^* \equiv \frac{t \mu}{D^2 \bar{\rho}} \Rightarrow t = \frac{t^* D^2 \bar{\rho}}{\mu}$$

$$\Rightarrow \frac{\partial v_z}{\partial t} = \frac{\partial \left( \frac{v_z^* \mu}{D \bar{\rho}} \right)}{\partial \left( \frac{t^* D^2 \bar{\rho}}{\mu} \right)} = \frac{\mu^2}{D^3 \bar{\rho}^2} \frac{\partial v_z^*}{\partial t^*}$$

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SOLUTION: z-component of the **nondimensional** Navier-Stokes Equation (free convection):

$$\frac{Dv_z^*}{Dt} = (\nabla^2 v_z)^* + \left[ \frac{g D^3 \bar{\rho}^2 \beta (T_2 - \bar{T})}{\mu^2} \right] T^*$$

Or any appropriate characteristic  $\Delta T$

Grashof number

$$(\nabla^2 v_z)^* \equiv \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( r^* \frac{\partial v_z^*}{\partial r^*} \right) + \frac{1}{r^{*2}} \frac{\partial^2 v_z^*}{\partial \theta^2} + \frac{\partial^2 v_z^*}{\partial z^{*2}}$$

$$\frac{Dv_z^*}{Dt} \equiv \left( \frac{\partial v_z^*}{\partial t^*} + v_r^* \frac{\partial v_z^*}{\partial r^*} + \frac{v_\theta^*}{r^*} \frac{\partial v_z^*}{\partial \theta} + v_z^* \frac{\partial v_z^*}{\partial z^*} \right)$$

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Dimensionless Equation of Motion (free convection)

$$\frac{Dv_z^*}{Dt^*} = (\nabla^2 v_z^*) + GrT^*$$

Dimensionless Energy Equation (free convection; Re = 1)

$$\left( \frac{\partial T^*}{\partial t^*} + \underline{v}^* \cdot \nabla^* T^* \right) = \frac{1}{Pr} \nabla^{*2} T^*$$

$$Nu = Nu\left(T^*, \frac{L}{D}\right) \Rightarrow Nu = Nu\left(Pr, Gr, \frac{L}{D}\right)$$

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**Example:** Natural convection from vertical planes and cylinders

$$Nu = \frac{hL}{k} = aGr^m Pr^m$$

- $a, m$  are given in Table 4.7-1, page 255 Geankoplis for several cases
- $L$  is the height of the plate
- all physical properties evaluated at the film temperature,  $T_f$

$$T_f = \frac{T_w + T_b}{2}$$

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