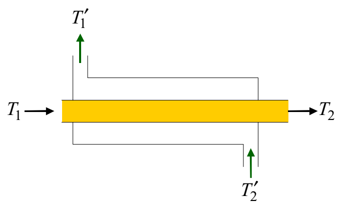
Heat Exchanger Effectiveness

To calculate Q, we need both inlet and outlet temperatures:

$$Q = UA\Delta T_m = UA\left(F_T \Delta T_{lm}\right)$$



$$\Delta T_{lm} = \frac{(T_1' - T_1) - (T_2' - T_2)}{\ln \frac{(T_1' - T_1)}{(T_2' - T_2)}}$$

What if the outlet temperatures are unknown? e.g. calculate the performance of a given heat exchanger.

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To calculate unknown outlet temperatures:

Procedure:
1. guess outlet temperatures
2. calculate ΔT_{lm}, F_T
3. calculate Q
4. calculate Q from energy balance
5. compare, adjust, repeat.

This tedious procedure can be simplified by the definition of heat-exchanger effectiveness, ε.

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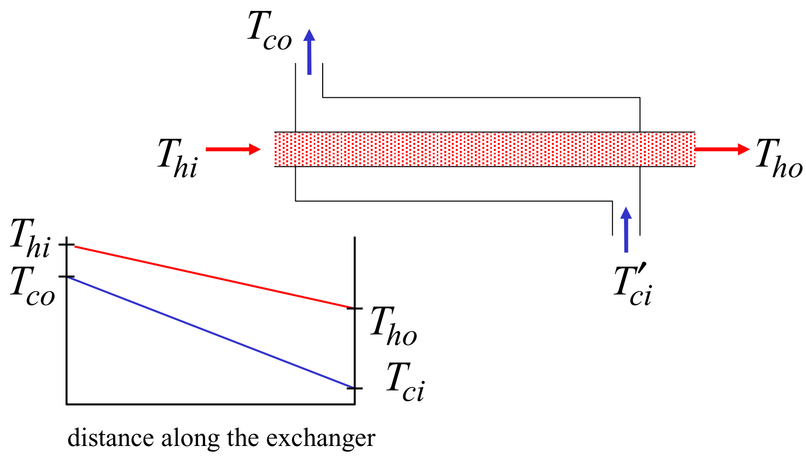
Heat Exchanger Effectiveness

Consider a *counter-current* double-pipe heat exchanger:



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Energy balance cold side:

$$Q_{in,cold} = Q = \left(mC_p\right)_{cold}\left(T_{co} - T_{ci}\right)$$

Energy balance hot side:

$$Q_{in,hot} = -Q = \left(mC_p\right)_{hot}\left(T_{ho} - T_{hi}\right)$$

$$\frac{\left(mC_p\right)_{hot}}{\left(mC_p\right)_{cold}} = \frac{\left(T_{co} - T_{ci}\right)}{-\left(T_{ho} - T_{hi}\right)} = \frac{\Delta T_c}{\Delta T_h}$$

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Temperature profile in a double-pipe heat exchanger:

$$\frac{T'-T}{T_1'-T_1} = e^{\alpha_0 x} \qquad \alpha_0 = 2\pi RU\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)$$

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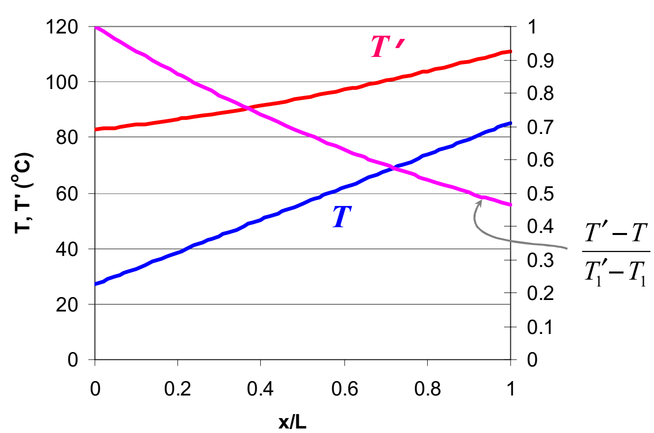
** REVIEW OF LECTURE 7 ** REVIEW OF LECTURE 7 **

Note that the temperature curves are only approximately linear.

$$\frac{T'-T}{T_1'-T_1} = e^{\alpha_0 L\left(\frac{x}{L}\right)}$$

$$\alpha_0 L = 2\pi RLU\left(\frac{1}{\hat{C}_p' m'} - \frac{1}{\hat{C}_p m}\right)$$



$U = 300\ W/mK$
$2\pi RL = 15.4 m^2$
$m' C_p' = 6\ kW/K$
$m C_p = 3\ kW/K$

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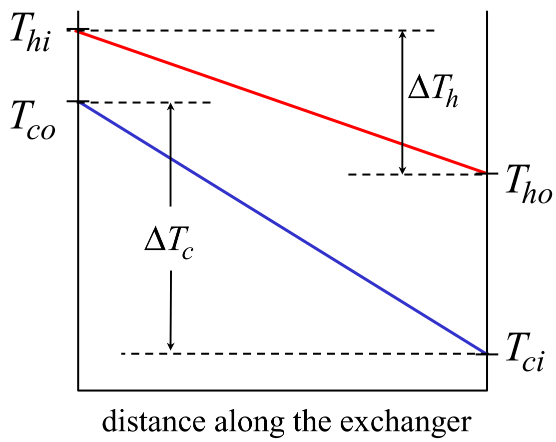
Case 1: $\begin{cases} (mC_p)_{hot} > (mC_p)_{cold} \\ \Delta T_c > \Delta T_h \end{cases}$ cold fluid = minimum fluid



We want to compare the amount of heat transferred in this case to the amount of heat transferred in a PERFECT heat exchanger.

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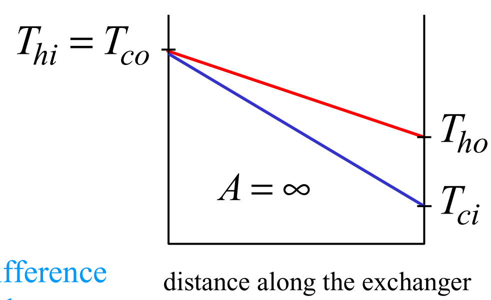
If the heat exchanger were *perfect*, $T_{hi}=T_{co}$



<u>cold side</u>:

this temperature difference only depends on inlet temperatures

$$Q_{A=\infty} = (mC_p)_{cold}\underbrace{(T_{hi} - T_{ci})}$$

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Heat Exchanger Effectiveness, ε

$$\varepsilon \equiv \frac{Q}{Q_{A=\infty}}$$

$$\Rightarrow Q = \varepsilon \left(mC_p\right)_{cold}\left(T_{hi} - T_{ci}\right)$$

cold fluid = minimum fluid

if ε is known, we can calculate Q
without iterations

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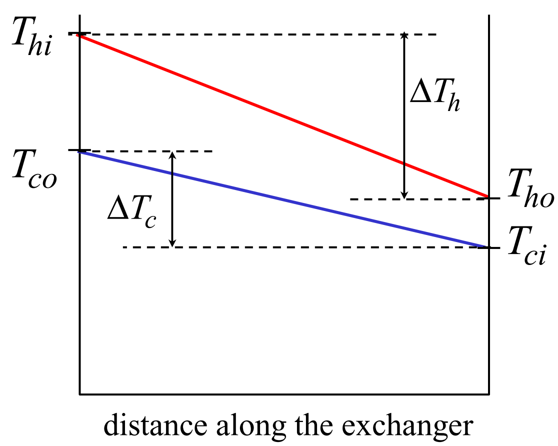
Case 2: $\begin{cases} \left(mC_p\right)_{hot} < \left(mC_p\right)_{cold} \\ \Delta T_c < \Delta T_h \end{cases}$ hot fluid = minimum fluid



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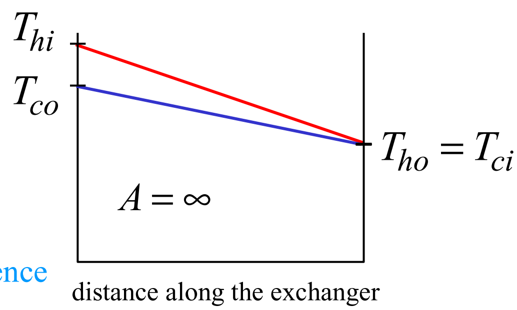
If the heat exchanger were *perfect*, $T_{hi}=T_{co}$



<u>hot side</u>:

this temperature difference only depends on inlet temperatures

$$Q_{A=\infty} = \left(mC_p\right)_{hot}\left(T_{hi} - T_{ci}\right)$$

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Heat Exchanger Effectiveness, ε

$$\varepsilon \equiv \frac{Q}{Q_{A=\infty}}$$

$$\Rightarrow Q = \varepsilon \left(mC_p\right)_{hot}\left(T_{hi} - T_{ci}\right)$$

hot fluid = minimum fluid

in general,

$$Q = \varepsilon \left(mC_p\right)_{\min}\left(T_{hi} - T_{ci}\right)$$

if ε is known, we can calculate Q without iterations

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But where do we get ε ?

The same equations we use in the trial-and-error solution can be combined algebraically to give ε as a function of $(mC_p)_{min}$, $(mC_p)_{max}$.

counter-current flow:

$$\varepsilon = \left(\frac{1 - e^{\frac{-UA}{(mC_p)_{\min}}\left(1 - \frac{(mC_p)_{\min}}{(mC_p)_{\max}}\right)}}{1 - \frac{(mC_p)_{\min}}{(mC_p)_{\max}} e^{\frac{-UA}{(mC_p)_{\min}}\left(1 - \frac{(mC_p)_{\min}}{(mC_p)_{\min}}\right)}} \right)$$

This relation is plotted in Geankoplis, as is the relation for co-current flow.

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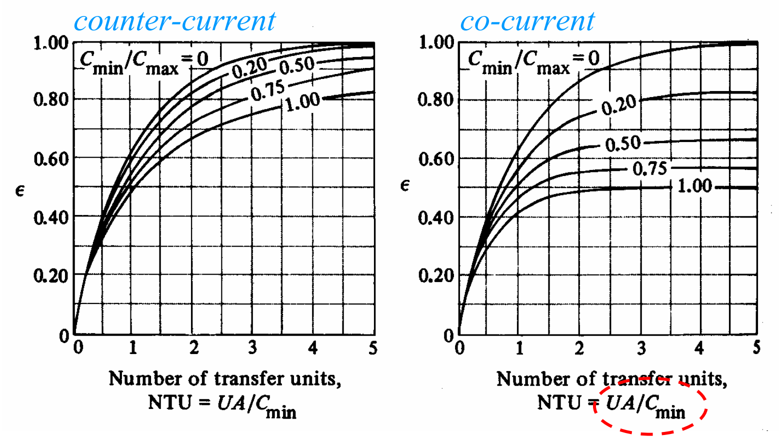
Heat Exchanger Effectiveness for Double-pipe or 1-1 Shell-and-Tube Heat Exchangers



Geankoplis 4th ed., p299

note: Geankoplis' $C_{min}=(mC_p)_{min}$

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Final Exam

CM 310

November 16, 1998

3. (25 points) Water flowing at a rate of 0.723 kg/s enters the inside of a countercurrent, double-pipe heat exchanger at 300K and is heated by an oil stream that enters at 385K at a rate of 3.2kg/s. The heat capacity of the oil is 1.89 kJ/kg K, and the average heat capacity of water over the temperature range of interest is 4.192 kJ/kg K. The overall heat-transfer coefficient of the exchanger is 300 W/m^2 K, and the area for heat transfer is 15.4 m^2. What is the total amount of heat transferred?

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