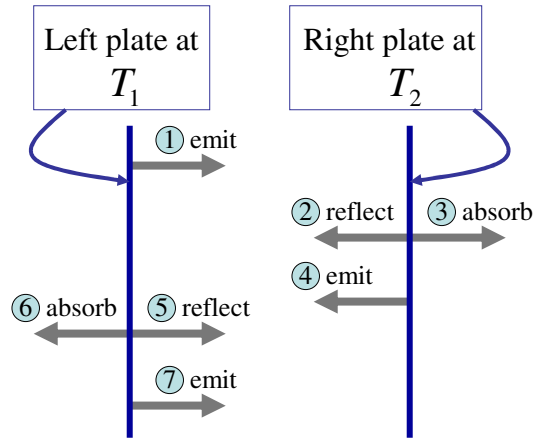


Radiation Heat Transfer Between Two Infinite Plates

Consider a quantity of radiation energy that is emitted from surface 1.



See: Geankoplis, section 4.11B

Also: Bird, Stewart, and Lightfoot, "Transport Phenomena" 1960 Wiley PP446-448

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First round – surface 2

Radiation Heat Transfer Between Two Infinite Plates

Quantity of energy **incident** at surface 2:

$$\frac{q_{1-2}}{A} = \epsilon_1 \sigma T_1^4$$

Quantity of energy **absorbed** at surface 2:

$$\alpha_2 \left(\frac{q_{1-2}}{A} \right) A = \epsilon_2 (\epsilon_1 \sigma T_1^4) A$$

$\alpha_2 = \epsilon_2$

Quantity of energy **reflected** from surface 2:

$$(1 - \epsilon_2) (\epsilon_1 A \sigma T_1^4)$$

fraction reflected incident energy

↑
This energy goes back to surface 1.

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Second round – surface 1

Radiation Heat Transfer
Between Two Infinite Plates

Quantity of energy
absorbed at surface 1
(second round):

$$\underbrace{\epsilon_1}_{\text{fraction absorbed}} \underbrace{\left[(1 - \epsilon_2) (\epsilon_1 A \sigma T_1^4) \right]}_{\text{incident energy}}$$

Quantity of energy
reflected from surface 1
(second round):

$$\underbrace{(1 - \epsilon_1)}_{\text{fraction reflected}} \underbrace{\left[(1 - \epsilon_2) (\epsilon_1 A \sigma T_1^4) \right]}_{\text{incident energy}}$$

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Third round – surface 2

Radiation Heat Transfer
Between Two Infinite Plates

Quantity of energy
absorbed at surface 2
(third round):

$$\underbrace{\epsilon_2}_{\text{fraction absorbed}} \underbrace{\left[(1 - \epsilon_1)(1 - \epsilon_2) (\epsilon_1 A \sigma T_1^4) \right]}_{\text{incident energy}}$$

Quantity of energy
reflected from surface 2
(third round):

$$\underbrace{(1 - \epsilon_2)}_{\text{fraction reflected}} \underbrace{\left[(1 - \epsilon_1)(1 - \epsilon_2) (\epsilon_1 A \sigma T_1^4) \right]}_{\text{incident energy}}$$

There is a pattern.

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Now, calculate the radiation energy going from surface 1 to surface 2:

Later, calculate energy from 2 to 1; then subtract to obtain **net energy transferred**.

$$\begin{aligned}
 q_{1 \rightarrow 2} &= \left(\begin{array}{c} \text{energy from} \\ 1 \rightarrow 2 \end{array} \right) = \sum \left(\begin{array}{c} \text{energy absorbed} \\ \text{at surface 2} \end{array} \right) \\
 &= \epsilon_2 (\epsilon_1 A \sigma T_1^4) \\
 &\quad + \epsilon_2 (1 - \epsilon_1)(1 - \epsilon_2) (\epsilon_1 A \sigma T_1^4) \\
 &\quad + \epsilon_2 (1 - \epsilon_1)^2 (1 - \epsilon_2)^2 (\epsilon_1 A \sigma T_1^4) \\
 &\quad \dots + \epsilon_2 (1 - \epsilon_1)^n (1 - \epsilon_2)^n (\epsilon_1 A \sigma T_1^4) + \dots
 \end{aligned}$$

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Radiation energy going from surface 1 to surface 2:

$$q_{1 \rightarrow 2} = \epsilon_1 \epsilon_2 A \sigma T_1^4 \sum_{n=0}^{\infty} (1 - \epsilon_1)^n (1 - \epsilon_2)^n$$

How can we calculate $\sum_{n=0}^{\infty} x^n$?

Answer: $S = \frac{1}{1-x}$

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Radiation energy going from
surface 1 to surface 2:

$$q_{1-2} = \frac{\epsilon_1 \epsilon_2 A \sigma T_1^4}{1 - [(1 - \epsilon_1)(1 - \epsilon_2)]}$$

$$= \frac{\epsilon_1 \epsilon_2 A \sigma T_1^4}{1 - [1 - \epsilon_1 - \epsilon_2 + \epsilon_1 \epsilon_2]} = \frac{\epsilon_1 \epsilon_2 A \sigma T_1^4}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$\frac{q_{1-2}}{A} = \frac{\sigma T_1^4}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Final Result

Radiation energy going from
surface 1 to surface 2:

$$\frac{q_{1-2}}{A} = \frac{\sigma T_1^4}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

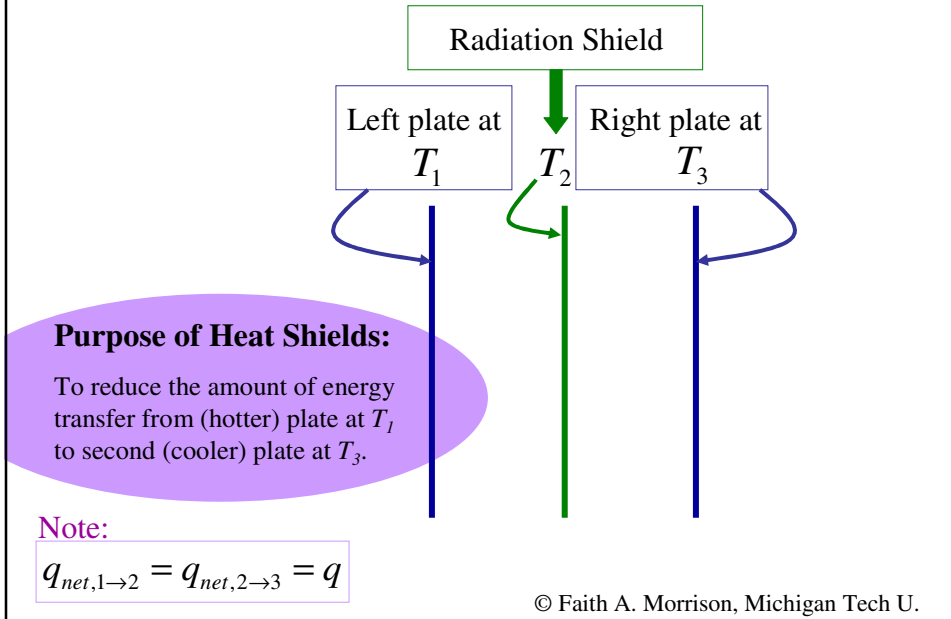
Radiation energy going from
surface 2 to surface 1:

$$\frac{q_{2-1}}{A} = \frac{\sigma T_2^4}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

NET Radiation energy going
from surface 1 to surface 2:

$$\frac{q_{1-2} - q_{2-1}}{A} = \frac{\sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right)}$$

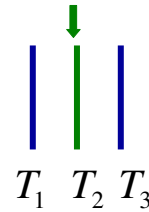
Radiation Shields



Analysis of Radiation Shields

We will assume that the emissivity is the same for all surfaces.

Radiation Shield



$$\frac{q_{net,1 \rightarrow 2}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1\right)}$$

$$\frac{q_{net,2 \rightarrow 3}}{A} = \frac{\sigma(T_2^4 - T_3^4)}{\left(\frac{1}{\epsilon} + \frac{1}{\epsilon} - 1\right)}$$

Now we eliminate T_2 between these equations.

Note:

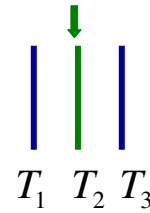
$$q_{net,1 \rightarrow 2} = q_{net,2 \rightarrow 3} = q$$

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Analysis of Radiation Shields

Radiation Shield

$$\frac{q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{2}{\epsilon} - 1\right)} \quad \frac{q}{A} = \frac{\sigma(T_2^4 - T_3^4)}{\left(\frac{2}{\epsilon} - 1\right)}$$



$$T_2^4 = \frac{q}{\sigma A} \left(\frac{2}{\epsilon} - 1\right) + T_3^4$$

$$\frac{q}{\sigma A} \left(\frac{2}{\epsilon} - 1\right) = T_1^4 - \frac{q}{\sigma A} \left(\frac{2}{\epsilon} - 1\right) - T_3^4$$

$$\frac{2q}{\sigma A} \left(\frac{2}{\epsilon} - 1\right) = T_1^4 - T_3^4$$

$$\frac{q}{A} = \left(\frac{1}{2}\right) \frac{\sigma(T_1^4 - T_3^4)}{(2/\epsilon - 1)}$$

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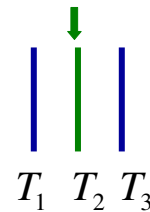
Analysis of Radiation Shields

Radiation Shield

1 Heat Shield

$$\frac{q}{A} = \left(\frac{1}{2}\right) \frac{\sigma(T_1^4 - T_3^4)}{(2/\epsilon - 1)}$$

With one heat shield present, q falls by half compared to no heat shield.



by the same analysis,

$$\frac{q}{A} = \left(\frac{1}{N+1}\right) \frac{\sigma(T_1^4 - T_3^4)}{(2/\epsilon - 1)}$$

With N heat shields present, q falls by a factor of $1/N$ compared to no heat shield.

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