What we know about Fluid Mechanics

1. MEB (single input, single output, steady, incompressible, no rxn, no phase change, little heat; good for pipes, pumps; Moody chart; Fanning friction factor versus Re)
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2. Fluid Statics ($P_{bot} = P_{top} + \rho gh$; same elevation, same pressure; good for manometers, water in tanks)

3. Math is in our future
How do fluids behave?

1. Viscosity
2. Drag
3. Boundary Layers
4. Laminar versus Turbulent Flow
5. Lift
6. Supersonic
7. Surface Tension
8. Curved Streamlines
9. Magnetohydrodynamics

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

2.1 Viscosity, $\mu$

A measure of a liquid's resistance to flow

water (modest viscosity) honey (high viscosity)
Viscous fluids transmit stress from one location to another.

Momentum Flux

Momentum \( p = \text{mass} \times \text{velocity} \)

Viscosity determines the magnitude of momentum flux

Vectors

top plate has momentum, and it transfers this momentum to the top layer of fluid
How is force to move plate related to $V$?

\[
\frac{F}{A} = +\mu \frac{V}{H} = \mu \left( \frac{v_z|_{y=0} - v_z|_{y=H}}{H - 0} \right) \quad \text{(Note choice of coordinate system)}
\]

Stress on a $y$-surface in the $z$-direction

\[
\hat{\tau}_{yz} = -\mu \left( \frac{\Delta v_z}{\Delta y} \right)
\]

Newton’s Law of Viscosity

\(\frac{\text{stress on a } y\text{-surface in the } z\text{-direction}}{\text{flux of } z\text{-momentum}}\)

Momentum Flux

9 stresses at a point in space

\[
\hat{\tau} = \frac{\text{force}}{\text{area}} = \frac{kg \ m / s^2}{\text{area}} = \frac{(kg) (m / s)}{(s)(\text{area})}
\]

(See discussion of sign convention of stress; we use the tension-positive convention)
Example 2.1: What are the units of viscosity?

\[ \tau_{yz} = \mu \left( \frac{dv_z}{dy} \right) \]

**Newton’s Law of Viscosity**

Viscosity, Greek letter “mu”

**Viscosity**

A measure of a liquid’s resistance to flow

Viscous fluids transmit stress from one location to another.

Viscosity is responsible for the development of pressure distributions in laminar flow.
Example 2.2: How much force does it take to inject a water-like solution through a 16-gauge needle (inner diameter=1.194 mm, \( L=40\text{mm} \))?

Need to know: \( \Delta p(Q) \)

From the methods of this course, we shall see that for Newtonian fluids:

Hagen-Poiseuille equation (slow flow through tubes)

\[
Q = \frac{\pi (p_0 - p_L) R^4}{8 \mu L}
\]
In the momentum balance, viscosity produces a force:

\[ \sum_{\text{all forces}} f = ma = \text{Rate of change of momentum} \]

**Forces**  
(including viscous forces)  

**Inertia**

2.2 Drag \( F_{\text{drag}} \)

The retarding force on an object due to a fluid (retarding implies opposite in direction to the fluid velocity)

**We study how to:**

*Calculate* drag,

1. Calculate velocity  
2. Calculate force on the object surface  
3. Calculate the component of that force in the direction of the flow

When impossible to calculate,

1. Measure force on model in a wind/water tunnel  
2. Correlate using dimensional analysis  
3. Scale up to system of interest

Drag is a consequence of viscosity
Without drag, objects of different weights, shapes, fall at the same speed:

In 1971, astronaut David Scott conducted Galileo’s experiment on the moon as part of Apollo 15.

Drag Coefficient, $C_D$

$$C_D \equiv \frac{F_{\text{drag}}}{\frac{1}{2} \rho \langle v \rangle^2 A_p}$$

True at high speeds

Reference area

Image from: www.seriouswheels.com

Image from: www.autoevolution.com
Example 2.4: How much faster will a bicycle racer traveling at 40 mph go if she adopts a racing crouch rather than riding upright?

- Upright: $C_D = 1.1$
- Racing crouch: $C_D = 0.88$
- Drafting: $C_D = 0.50$
- Recumbent: $C_D = 0.12$

Under what conditions is drag a simple matter of knowing $C_D$?

Could vary with:
- Flow speed
- Shape
- Density
- Viscosity
- Temperature
- ...

$$C_D \equiv \frac{F_{\text{drag}}}{\frac{1}{2} \rho \langle v \rangle^2 A_p}$$

(i.e., Why is this so?
When is this so?)
Drag behavior of a sphere

(Similar to using the Moody Chart when interested in wall drag in pipes)

Moody Chart: Data Correlation for Friction in Straight Pipes

(image from: Geankoplis)
2.3 Boundary Layers

• Regions near solid surfaces in which viscosity dominates the flow behavior, especially at high speeds.

There is relative motion near the surfaces (viscosity is important);

Away from surfaces, the flow is uniform (viscosity is not important; inertia dominates).
Why is the surface of a golf ball designed the way it is?


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Manipulate boundary-layer separation

When the boundary layer is turbulent, it detaches farther back (yielding lower drag)

smooth ball rough ball

H. Schlichting, Boundary Layer Theory (McGraw-Hill, NY 1955.)

cover: This image from a simulation of wind blowing past a building (black square) reveals the vortices that are shed downwind of the building; dark orange represents the highest air speeds, dark blue the lowest. As a result of such vortex formation and shedding, tall buildings can experience large, potentially catastrophic forces. (Courtesy of the computational fluid dynamics group at Rowan William Davies and Irwin Inc.)
Example 2.5: A new tower hotel, cylindrical in shape and 100 ft in diameter, has been built in a resort town near the sea on the windward side of an island. Hotel guests complain that there are often uncomfortably high winds near several of the entrances to the tower. How do the wind speed and pressure vary with position around the tower and with on-shore wind speed?

• The full Navier-Stokes is hard to solve; when viscosity is zero, however, it’s easy to solve the N-S for \( \mathbf{v} \) and \( p \)

• Viscosity is NOT zero; however, outside the boundary layer, viscosity is not important.

• STRATEGY: When away from surfaces, solve for outer (viscosity=0) flow

For inviscid flows:

• \( \mathbf{v} \) comes from stream function, \( \psi' \); (Diff Eqns)

• Pressure comes from \( \mathbf{v} \) and the Bernoulli equation:

Bernoulli equation

\[
\frac{p}{\rho} + \frac{\langle \mathbf{v} \rangle^2}{2} + z = \text{constant along a streamline}
\]
Boundary Layers

Velocity outside boundary layer

cylinder in uniform flow:

\[
\begin{align*}
\nu &= \begin{cases} 
U \left(1 - \frac{R^2}{r^2}\right) \cos \theta \\
-U \left(1 + \frac{R^2}{r^2}\right) \sin \theta \\
0
\end{cases} 
\end{align*}
\]

(potential flow solution)

Pressure outside boundary layer

Bernoulli equation

\[
\frac{\rho}{\rho} + \frac{v^2}{2} + z = \text{constant along a streamline}
\]

Boundary Layers in Internal Flow:

Entrance flow field in pipe flow

Entrance region

Boundary layer

Fully developed flow
2.4 Laminar versus Turbulent Flow

Viscosity dominates

Inertia dominates

Reynolds’ Experiment

Viscosity dominates

Inertia dominates

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Take-Away from Today:
How do fluids behave?

1. Viscosity
   \[ \tau_{yz} = \mu \left( \frac{dv}{dy} \right) \]

2. Drag
   \[ C_D = \frac{F_{\text{drag}}}{\frac{1}{2} \rho \langle v \rangle^2 A_p} \]

3. Boundary Layers
   Viscous effects within BL; no viscous effects in main stream; Bernoulli equation (like MEB) assumes no viscous effects
   \[ \left( \text{outside the boundary layer} \right) \frac{p}{\rho} + \frac{v^2}{2} + z = \text{constant along a streamline} \]

4. Laminar versus Turbulent Flow
   (we know about this already)
Take-Away from Today:
How do fluids behave?

1. Viscosity
\[ \tau_{yz} = \mu \frac{dv}{dy} \]

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\[ C_D = \frac{F_{\text{drag}}}{\frac{1}{2} \rho \langle v \rangle^2 A_p} \]

3. Boundary Layers
Viscous effects within BL; no viscous effects in main stream;
Bernoulli equation (like MEB) assumes no viscous effects

(inside the boundary layer)
\[ \left( \frac{p}{\rho} + \frac{v^2}{2} + z \right) = \text{constant along a streamline} \]

(outside the boundary layer)

4. Laminar versus Turbulent Flow
(we know about this already)

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3. Newton’s Law of Viscosity (fluids transmit forces through momentum flux)
4. Momentum flux (=stress) has 9 components
5. Drag is a consequence of viscosity
6. Boundary layers form (viscous effects are confined near surfaces at high speeds)

8. Sometimes viscous effects dominate; sometimes inertial effects dominate
Need one more tool: \textbf{Control Volume} (Ch3)

Following fluid particles is complex:

It is simpler to observe the flow pass through a fixed volume

\textbf{Control Volume}

A chosen volume in a flow on which we perform balances (mass, momentum, energy)

• Shape, size are arbitrary; choose to be convenient

• Because we are now balancing on \textit{control volumes} instead of on \textit{bodies}, the laws of physics are written differently
Mass balance, flowing system (open system; control volume):

\[
\begin{align*}
\left\{ \text{net mass flowing in} \right\} &= \left\{ \text{rate of accumulation of mass} \right\} \\
\sum_{in} - \sum_{out} &= \text{steady state}
\end{align*}
\]

Momentum balance, flowing system (open system; control volume):

\[
\begin{align*}
\sum_i \left\{ \text{momentum flowing in} \right\} - \sum_i \left\{ \text{momentum flowing out} \right\} &= 0 \\
\sum F_{on} + \sum_i \left\{ \text{momentum flowing in in the streams} \right\} &= \sum_i \left\{ \text{momentum flowing out in the streams} \right\}
\end{align*}
\]

\[\sum_{all \ forces} f = ma \Rightarrow \]

Note that momentum is a vector quantity.
We are ready to try a momentum balance.

**Tools:**

- Mass balance (mass conserved)
- Newton’s 2nd law (momentum conserved)
- Control volume
- Newton’s law of viscosity
- Calculus 3 (multivariable calculus)