



Michigan Tech

CM3110**Transport I****Part I: Fluid Mechanics: *Microscopic Balances*****Professor Faith Morrison**

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Michigan Technological University

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What we know about Fluid Mechanics

1. MEB (single input, single output, steady, incompressible, no rxn, no phase change, little heat; good for pipes, pumps; Moody chart; Fanning friction factor versus Re)
2. Fluid Statics ($P_{bot} = P_{top} + \rho gh$); same elevation, same pressure; good for manometers, water in tanks)
3. Math is in our future
4. Newton's Law of Viscosity (fluids transmit forces through momentum flux)
5. Momentum flux (=stress) has 9 components
6. Drag is a consequence of viscosity
7. Boundary layers form (viscous effects are confined near surfaces at high speeds)
8. Sometimes viscous effects dominate; sometimes inertial effects dominate
- 9.
- 10.
- 11.

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What we know about Fluid Mechanics

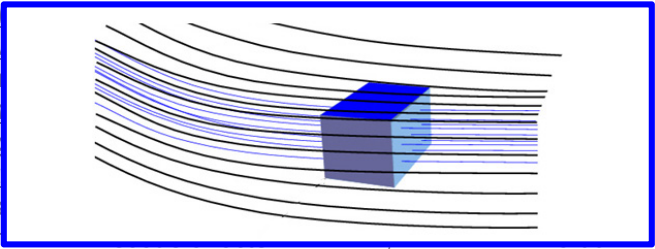
Newton's second law applies: $\underline{f} = m\underline{a}$

1. MEE rxn, Moo
2. Fluid good for manometers, water in tanks)
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9. **Momentum balance determines velocity distributions**
- 10.

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What we know about Fluid Mechanics

Newton's second law applies: $\underline{f} = m\underline{a}$



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6. Drag
7. Bou surf
8. Som effects dominate
9. **Momentum balance determines velocity distributions**
10. **Control volumes are valuable for balances in fluids**

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Following a Solid Object

(PH2100)

$$\underline{f} = m\underline{a}$$

- Forces on **ball**
- Mass of **ball**
- Acceleration of **ball**

Image from www.ux1.eiu.edu/~cfadd/1350/09Mom/CoM.html

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Following fluid particles is complex:

(x₀, y₀, z₀)

An Essential Tool: Control Volume (Ch3)

fluid particle pathlines

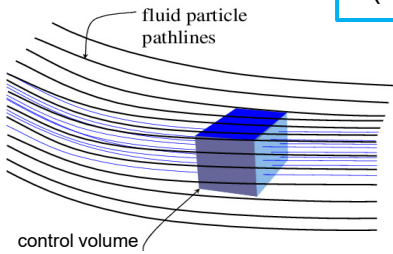
control volume

It is simpler to observe the flow pass through a fixed volume, than to follow fluid particles

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Control Volume

A chosen volume in a flow on which we perform balances (mass, momentum, energy)



fluid particle pathlines


control volume

- Shape, size are arbitrary; choose to be convenient
- Because we are now balancing on *control volumes* instead of on *bodies*, the laws of physics are written differently

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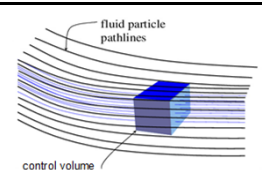
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Mass balance, *body*: $M_{body} = \text{constant}$



$$\frac{dM_{body}}{dt} = 0$$

Mass balance, flowing system (open system; *control volume*):



fluid particle pathlines

control volume

Convective term

net mass flowing in

$$\sum_{in} - \sum_{out}$$

= ~~rate of accumulation of mass~~

steady state

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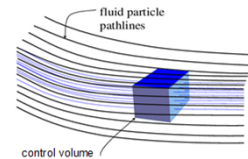
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Momentum balance, **body**:



$$\sum_{\text{on body}} \underline{f} = M_{\text{body}} \underline{a}$$

Momentum balance, flowing system
(open system; **control volume**):



Convective term

$$\left\{ \begin{array}{l} \text{sum of forces} \\ \text{acting on control vol} \end{array} \right\} + \left\{ \begin{array}{l} \text{net momentum} \\ \text{flowing in} \\ \sum_{in} - \sum_{out} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of} \\ \text{accumulation} \\ \text{of momentum} \end{array} \right\}$$

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Momentum balance, flowing system
(open system; **control volume**):

$$\left\{ \begin{array}{l} \text{sum of forces} \\ \text{acting on control vol} \end{array} \right\} + \left\{ \begin{array}{l} \text{net momentum} \\ \text{flowing in} \\ \sum_{in} - \sum_{out} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of} \\ \text{accumulation} \\ \text{of momentum} \end{array} \right\}$$

steady state

$$\sum_j \underline{F}_{on_j} + \sum_i \left\{ \begin{array}{l} \text{momentum} \\ \text{flowing in} \\ \text{in the streams} \end{array} \right\} - \sum_i \left\{ \begin{array}{l} \text{momentum} \\ \text{flowing out} \\ \text{in the streams} \end{array} \right\} = 0$$

note that **momentum** is a vector quantity

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We are ready to try a momentum balance

Tools:

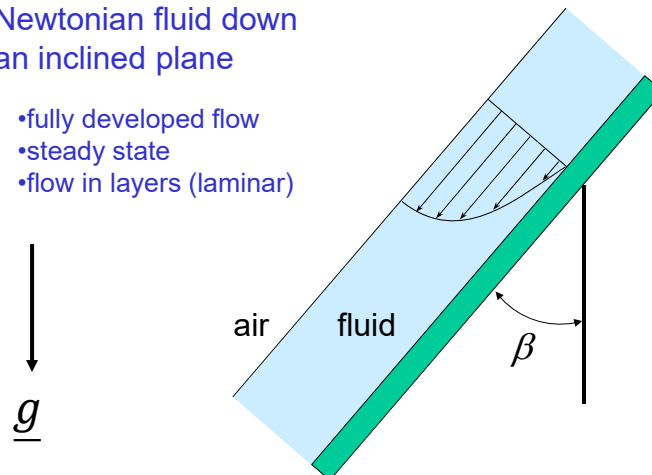
- Mass balance (mass conserved)
- Newton's 2nd law (momentum conserved)
- Control volume (convective term)
- Newton's law of viscosity
- Calc 2, Calc 3, Differential Eqns

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EXAMPLE 1: Flow of a Newtonian fluid down an inclined plane

- fully developed flow
- steady state
- flow in layers (laminar)



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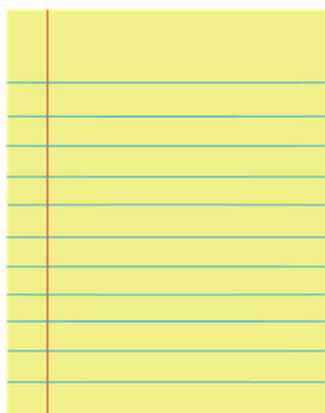
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EXAMPLE 1: Flow of a Newtonian fluid down an inclined plane

What is the velocity field in the steady flow of water down a slope that is wide and long. The fluid properties are constant, and the flow is driven by gravity. The flow is slow so that no waves are formed. What is the force on the surface due to the water flow? What is the flow rate?

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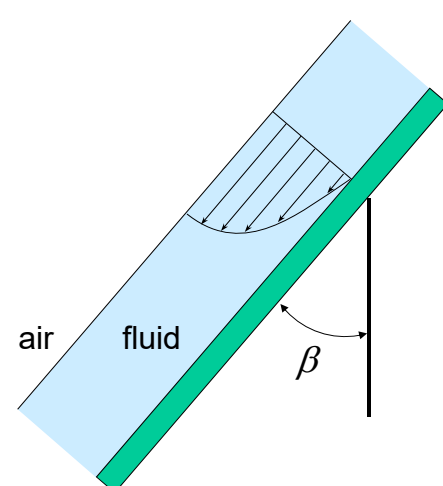
Where do we
start?

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1. Sketch problem

2.

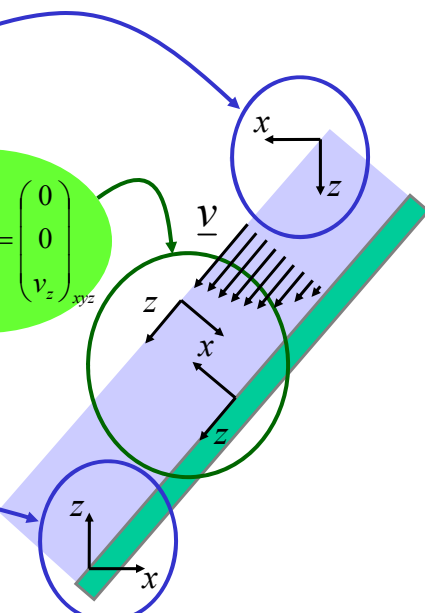


What next?

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$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz} = \begin{pmatrix} v_x \\ 0 \\ v_z \end{pmatrix}_{xyz}$$

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}_{xyz} = \begin{pmatrix} 0 \\ 0 \\ v_z \end{pmatrix}_{xyz}$$



Choose a coordinate system for convenience

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Gravity
(In chosen coordinate system)

$v_z(x)$

air fluid

β

z x

β $g_z = g \cos \beta$

\underline{g} **You try.**

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Gravity
(In chosen coordinate system)

$v_z(x)$

air fluid

β

z x

β $g_x = g \sin \beta$

$g_z = g \cos \beta$

\underline{g}

$$\underline{g} = \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix}_{xyz} = \begin{pmatrix} g \sin \beta \\ 0 \\ g \cos \beta \end{pmatrix}_{xyz}$$

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1. Sketch problem
2. Coordinate sys
- 3.

What next?

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Choose a convenient control volume

Want it to:

- Lead to what we're looking for
- Be easy to work with

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Choose a convenient control volume

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1. Sketch problem
2. Coordinate sys
3. Control volume
- 4.

What next?

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Recall:**Tools:**

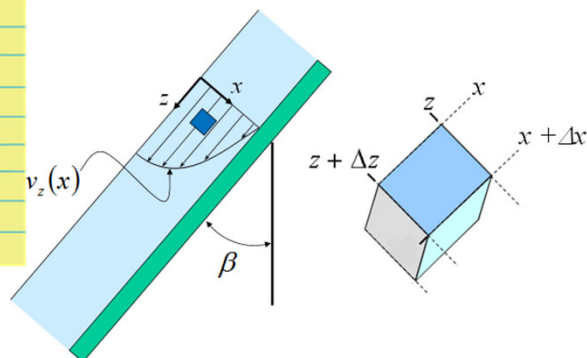
- Mass balance (mass conserved)
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Solution steps

1. Sketch problem
2. Coordinate sys
3. Control volume
4. Mass balance
5. Momentum bal
6. Solve
7. Plot

Let's do it.

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Mass balance, flowing system
(open system; **control volume**):

$$\left\{ \begin{array}{l} \text{net mass} \\ \text{flowing in} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of} \\ \text{accumulation} \\ \text{of mass} \end{array} \right\}$$

$$\sum_{in} - \sum_{out} \quad \text{steady state}$$

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Momentum balance, flowing system $\sum_{all\ forces} \underline{f} = m\underline{a} \Rightarrow$
(open system; **control volume**):

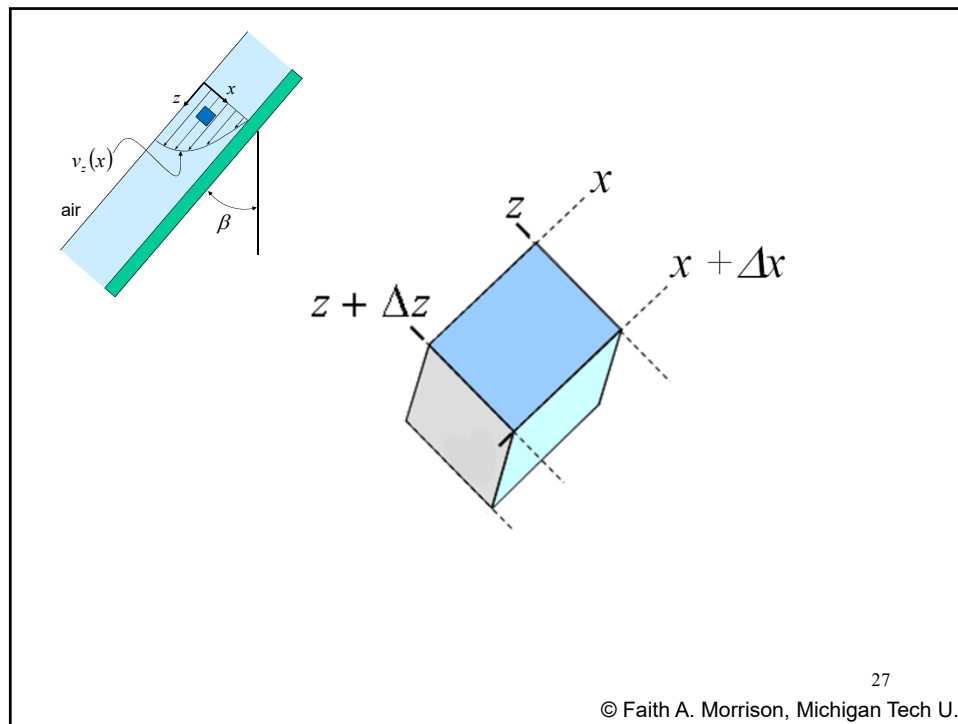
$$\left\{ \begin{array}{l} \text{sum of forces} \\ \text{acting on control vol} \end{array} \right\} + \left\{ \begin{array}{l} \text{net momentum} \\ \text{flowing in} \\ \sum_{in} - \sum_{out} \end{array} \right\} = \left\{ \begin{array}{l} \text{rate of} \\ \text{accumulation} \\ \text{of momentum} \end{array} \right\}$$

steady state

$$\sum_i \underline{F}_{on_i} + \sum_i \left\{ \begin{array}{l} \text{momentum} \\ \text{flowing in} \\ \text{in the streams} \end{array} \right\} - \sum_i \left\{ \begin{array}{l} \text{momentum} \\ \text{flowing out} \\ \text{in the streams} \end{array} \right\} = 0$$

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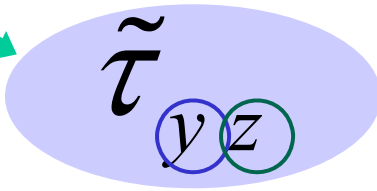
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See handwritten notes.

$\tilde{\tau}_{yz} = \frac{\text{force}}{\text{area}} = \frac{\text{kg m / s}^2}{\text{area}} = \frac{(\text{kg})(\text{m / s})}{(\text{s})(\text{area})}$

9 stresses at a point in space



$\tilde{\tau}$

y z

stress on a y-surface \rightarrow in the z-direction

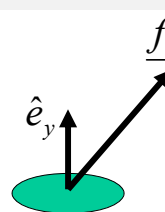
A surface whose unit normal is in the y-direction

in the y-direction \leftarrow flux of z-momentum

$-\tilde{\tau}_{yz} =$

(See discussion of sign convention of stress; this is the tension positive convention)

Momentum Flux



$\underline{f} = A(\tau_{yx}\hat{e}_x + \tau_{yy}\hat{e}_y + \tau_{yz}\hat{e}_z)$

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$\tau_{yz} = \mu \left(\frac{dv_z}{dy} \right)$

Newton's Law of Viscosity

(Scalar relationship; specific to one coordinate system)

$\underline{\underline{\tau}} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}_{xyz}$

\rightarrow

We will discuss the general case later

$\underline{\underline{\tau}} = \mu \underline{\underline{\dot{\gamma}}} = \mu(\nabla \underline{v} + (\nabla \underline{v})^T)$

Newtonian Constitutive Equation

(Tensor relationship; all coordinate systems)

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$$\tau_{xz} = \mu \left(\frac{dv_z}{dx} \right) \quad \text{Newton's Law of Viscosity}$$

(adapted to our coordinate system)

(Scalar relationship; specific to one particular coordinate system)

Newton's law gives the link between:

- *Deformation* (change of shape), and
- *Shear Stress* (shear force per area)

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Flow down an Incline Plane

Boundary conditions:

$$x = 0 \quad \tilde{\tau}_{xz} = 0$$

-stress matches at boundary

$$x = H \quad v_z = 0$$

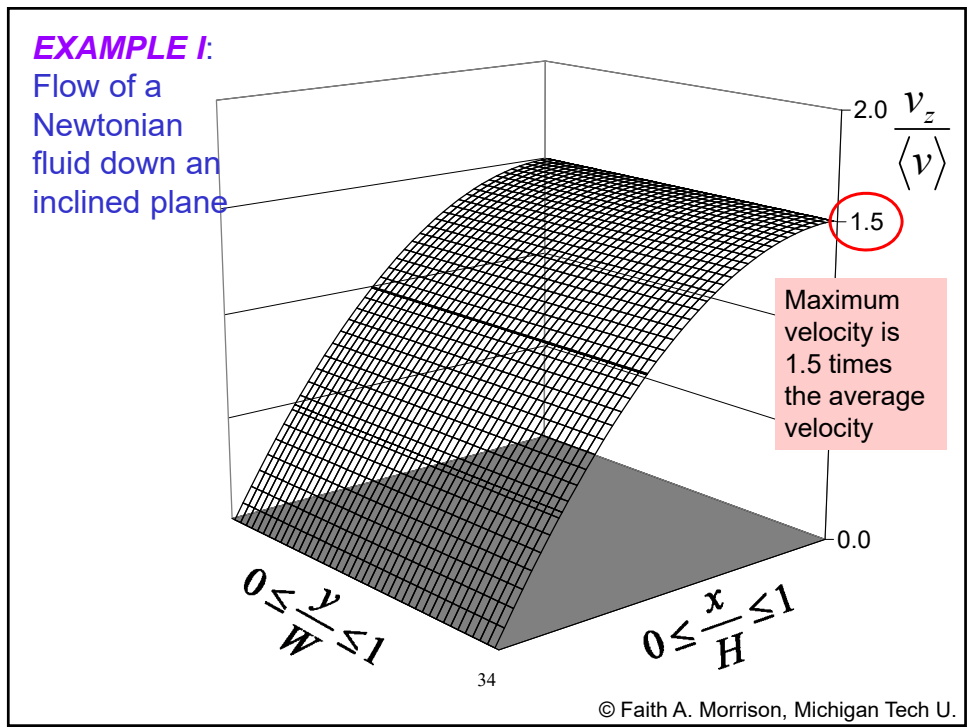
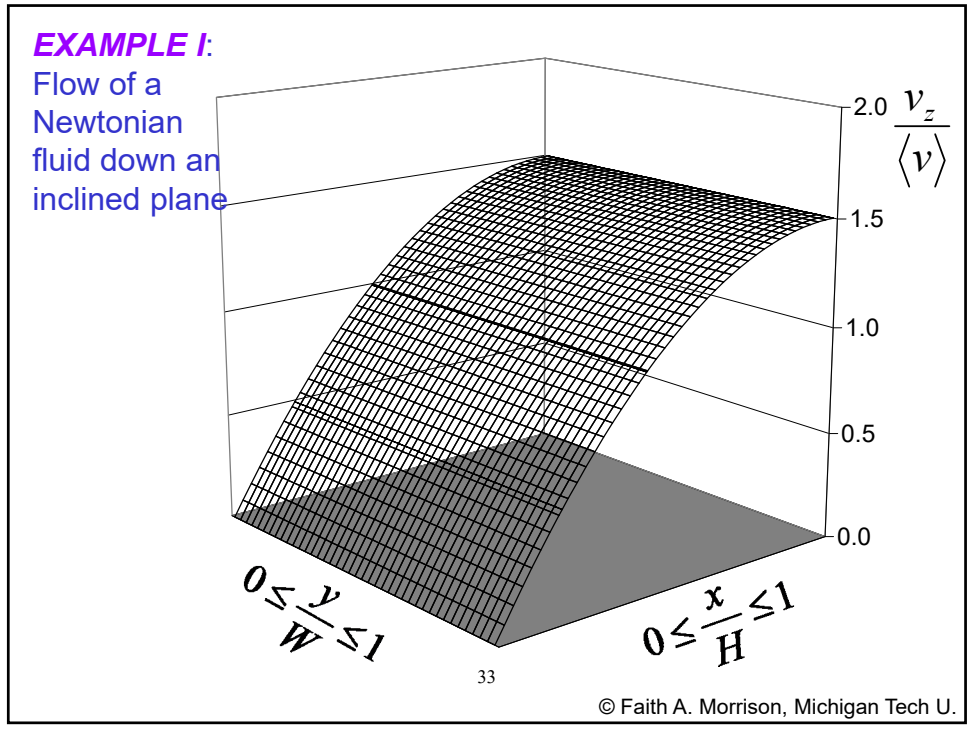
-no slip at the wall

Solution:

$$v_z(x) = \frac{\rho g \cos \beta}{2\mu} (H^2 - x^2)$$

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From the start of the problem, we developed our **model** step by step. We can collect our modeling assumptions, which are limitations on the result.

Model Assumptions: (laminar flow down an incline, Newtonian)

1. no velocity in the x- or y-directions (laminar flow)
2. well developed flow
3. no edge effects in y-direction (width)
4. constant density
5. steady state
6. Newtonian fluid
7. no shear stress at interface
8. no slip at wall

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Calculate: What is the shear stress as a function of position for this flow?

Newton's Law of Viscosity

$$\tilde{\tau}_{xz} = \mu \frac{\partial v_z}{\partial x}$$

You try.

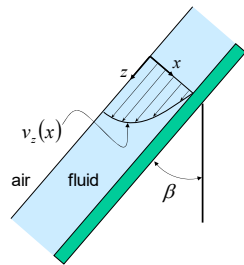
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We can now calculate:

Engineering Quantities of Interest

- Average velocity
- Volumetric flow rate
- Force on the wall



$$v_z(x) = \frac{\rho g \cos \beta}{2\mu} (H^2 - x^2)$$

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Engineering Quantities of Interest

average velocity

$$\langle v_z \rangle \equiv \frac{\int_0^W \int_0^H v_z \, dx \, dy}{\int_0^W \int_0^H dx \, dy}$$

H is the height of the film; W is the width

volumetric flow rate

$$Q = \int_0^W \int_0^H v_z \, dx \, dy = WH \langle v_z \rangle$$

z-component of force on the wall

$$F_z = \int_0^L \int_0^W \tilde{\tau}_{xz} \Big|_{x=H} \, dy \, dz$$

(The expressions are different in different coordinate systems)

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Engineering Quantities of Interest

(any flow)

volumetric
flow rate

$$Q = \iint_S (\hat{n} \cdot \underline{v}) dS$$

average
velocity

$$\langle v_z \rangle \equiv \frac{\iint_S (\hat{n} \cdot \underline{v}) dS}{\iint_S dA} = \frac{Q}{S}$$

z-component
of force on
the wall

$$F_z = \hat{e}_z \cdot \iint_S [\hat{n} \cdot (-p\underline{I} + \underline{\tilde{\tau}})]_{surface} dS$$

For more complex flows, we use the **Gibbs notation**
versions (will discuss soon).

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What did we do?

Problem-Solving Procedure – solving for velocity and stress fields

1. sketch system
2. choose coordinate system
3. choose a control volume
4. perform a mass balance
5. perform a momentum balance
(will contain stress)
6. substitute in *Newton's law of viscosity*, e.g.
7. solve the differential equation
8. apply boundary conditions

$$\tilde{\tau}_{xz} = \mu \left(\frac{dv_z}{dx} \right)$$

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How can we generalize this soln process?

(to make the process easier)

Answer: Develop a balance that works for all control volumes



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