Transport/Unit Operations

Professor Faith A. Morrison
Department of Chemical Engineering
Michigan Technological University

CM2120—Fundamentals of ChemE 2 (Steady Unit Operations Introduction, MEB)
CM3110—Transport/Unit Ops 1 (Momentum & Steady Heat Transport, Unit Operations)
CM3120—Transport/Unit Ops 2 (Unsteady Heat Transport, Mass Transport, Unit Operations

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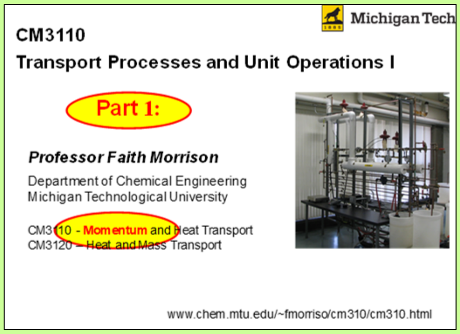
First section of the course is ***complete***.



CM3110
Transport Processes and Unit Operations I

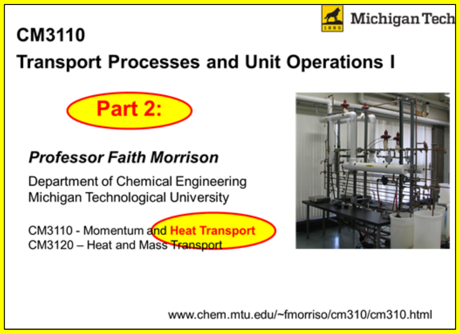
Part 1:

Professor Faith Morrison
Department of Chemical Engineering
Michigan Technological University

CM3110 - Momentum and Heat Transport
CM3120 – Heat and Mass Transport

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

Now we move on to the second part.



CM3110
Transport Processes and Unit Operations I

Part 2:

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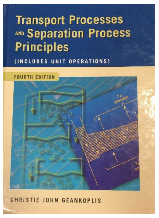
CM3110
Transport I
Part II: Heat Transfer



Introduction to Heat Transfer

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University

www.chem.mtu.edu/~fmorriso/cm310/cm310.html

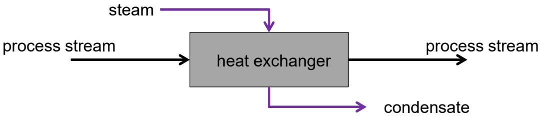
Where do we start?

Why are chemical engineers interested in heat transfer?

As before with fluid mechanics, there are engineering quantities of interest that can only be determined once we understand the physics behind heat transfer.

The physics of heat transfer is based on the **1st Law of Thermodynamics**: ***Energy is Conserved.***

Where do we start?



Where do we start?

Let’s start here:

How does ***energy*** behave?

1. Conduction (Brownian process)
2. Convection, Forced and Free/Natural (moves with moving matter)
3. Radiation (carried by electromagnetic waves)
4. Byproduct of Chemical Reaction
5. Byproduct of Electrical Current
6. Boundary layers (thermal)
7. Is a byproduct of Pressure-Volume Work (compressibility)
8. Is a byproduct of Viscous Dissipation
9. Simultaneous heat and mass transfer

Where do we start?

How does ***energy*** behave?

Often it is not possible to isolate heat transfer from other physics

Simultaneous heat and momentum transfer

1. Conduction (Brownian process)
2. Convection, Forced and Free/Natural (moves with moving matter)
3. Radiation (carried by electromagnetic waves)
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Where do we start?

How does ***energy*** behave?

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Advanced

Engineering Quantities of Interest

What do we want to determine?

From (momentum) we calculated:

Engineering Quantities of Interest (fluids)

- Average velocity
- Volumetric flow rate
- Force on a surface

From (energy) we calculate:

Engineering Quantities of Interest (energy)

- Shaft Work
- Total heat transferred
- Rate of heat transfer

Engineering Quantities of Interest

What do we want to determine?

From (momentum) we calculated:

Engineering Quantities of Interest (fluids)

- Average velocity
- Volumetric flow rate
- Force on a surface

And $\underline{v}$ & $\underline{\tilde{\tau}}$ distributions, which through dimensional analysis, lead to data correlations for complex systems, including chemical engineering unit ops

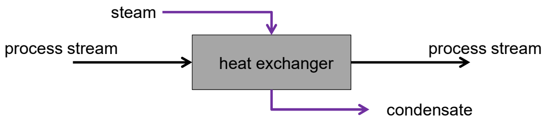
From (energy) we calculate:

Engineering Quantities of Interest (energy)

- Shaft Work
- Total heat transferred
- Rate of heat transfer

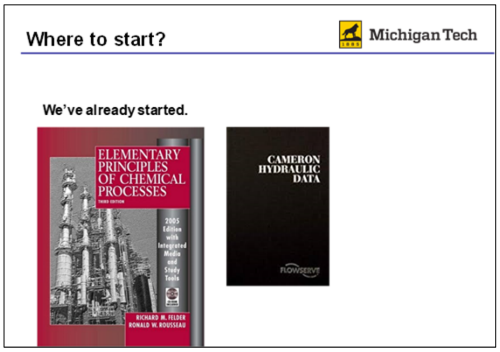
And T distributions, which … (see above)

Where do we start?



We've already started.

Recall from last year and earlier in the semester:



Where to start?

Michigan Tech

We've already started.

Energy** quantities of interest* ***Recall from CM2110

Steady State Macroscopic Energy Balances

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on} \qquad \text{(final-initial)}$$

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on} \qquad \text{(out-in)}$$

- Multiple inlets and outlets ($\Delta = \Sigma_{outs} - \Sigma_{ins}$)
- Steady state
- Constant density

Review:

www.chem.mtu.edu/~fmorriso/cm310/Energy_Balance_Notes_2008.pdf

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Recall from CM2110

Closed system energy balance

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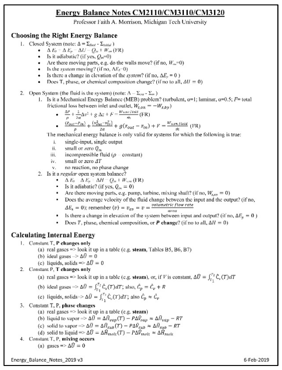

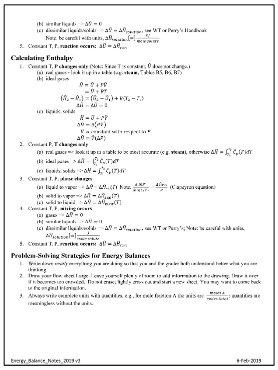

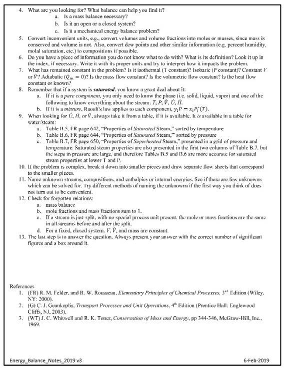

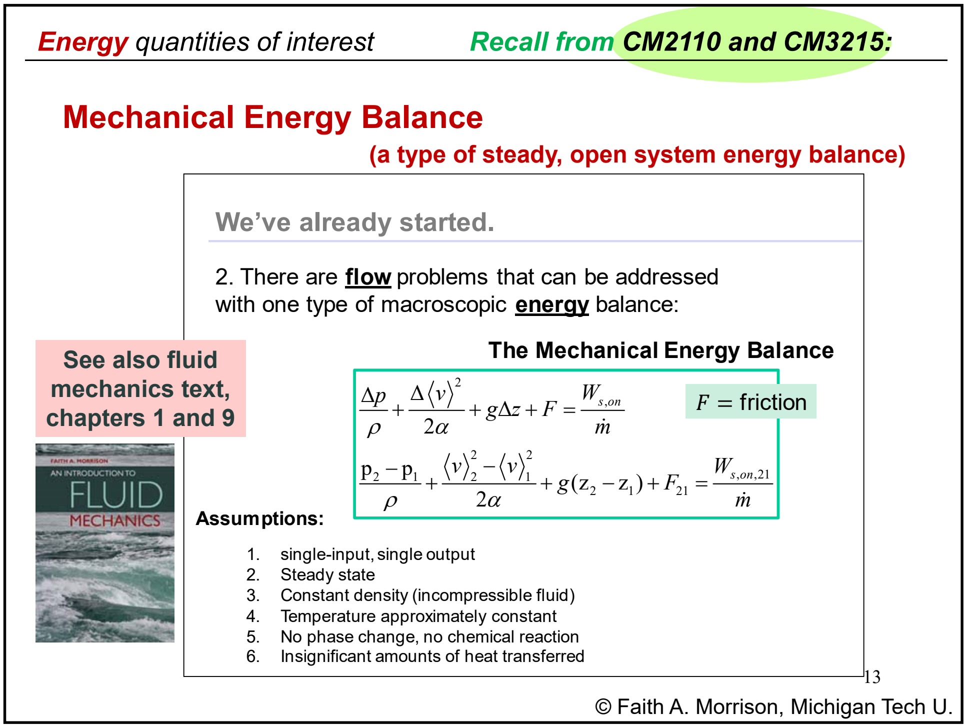

Energy** quantities of interest* ***Recall from CM2110 and CM3215:

Mechanical Energy Balance

(a type of steady, open system energy balance)

We've already started.

2. There are **flow** problems that can be addressed with one type of macroscopic **energy** balance:

The Mechanical Energy Balance

$$\frac{\Delta p}{\rho}+\frac{\Delta\langle v\rangle^2}{2\alpha}+g\Delta z+F=\frac{W_{s,on}}{\dot{m}}$$

F = friction

$$\frac{\mathrm{p}_2-\mathrm{p}_1}{\rho}+\frac{\langle v\rangle_2^2-\langle v\rangle_1^2}{2\alpha}+g(\mathrm{z}_2-\mathrm{z}_1)+F_{21}=\frac{W_{s,on,21}}{\dot{m}}$$

See also fluid mechanics text, chapters 1 and 9

Assumptions:

1. single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical reaction
6. Insignificant amounts of heat transferred

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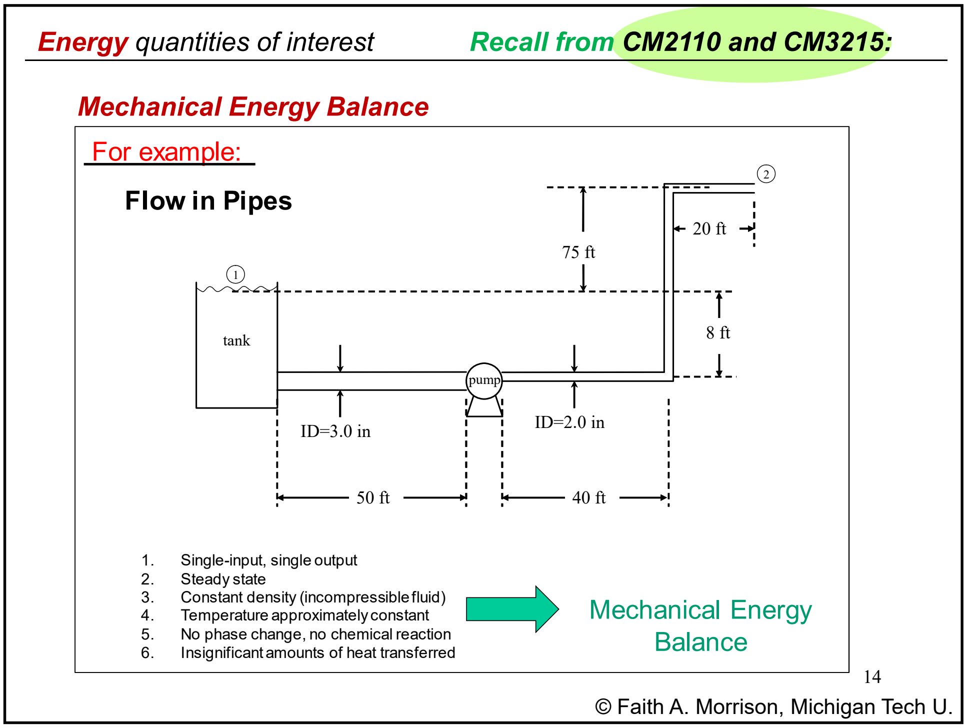

Energy** quantities of interest* ***Recall from CM2110 and CM3215:

Mechanical Energy Balance

For example:

Flow in Pipes

1. Single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical reaction
6. Insignificant amounts of heat transferred

Mechanical Energy Balance

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***Energy** quantities of interest* **Recall from CM2110 and CM3215:**

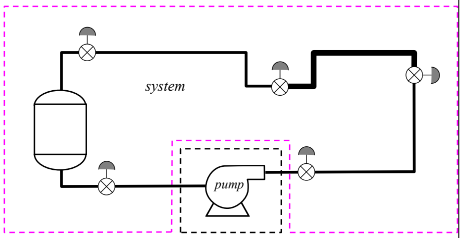
Mechanical Energy Balance

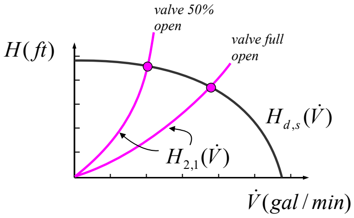
For example:

Centrifugal Pumps

What flow rate does a centrifugal pump produce?

Answer: Depends on how much work it is asked to do.





Calculate with the Mechanical Energy Balance (CM2110, CM2120, CM3215)

***Energy** quantities of interest* **Recall from CM2110 and CM3215:**

Mechanical Energy Balance

We can apply the MEB to many important engineering systems

Image from: www.directindustry.com

Image from: www.processindustryforum.com

Image from: www.directindustry.com

Image from: www.epa.gov

MEB Assumptions:

1. single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical rxn
6. Insignificant amounts of heat transferred

Calculate: **Work, pressures, flows**

Energy *quantities of interest* ***Recall from CM2110 and CM3215:***

Mechanical Energy Balance

The Mechanical Energy Balance

(Review)

$$\frac{\Delta p}{\rho}+\frac{\Delta\langle v\rangle^2}{2\alpha}+g\Delta z+F=\frac{W_{s,on}}{\dot{m}}$$

F = friction

$$\frac{p_2-p_1}{\rho}+\frac{\langle v\rangle_2^2-\langle v\rangle_1^2}{2\alpha}+g(z_2-z_1)+\boxed{F_{21}}=\frac{W_{s,on,21}}{\dot{m}}$$

Where do we get this?

This is the friction due to wall drag (straight pipes) and fittings and valves.

Energy *quantities of interest* ***Recall from CM2110 and CM3215:***

Mechanical Energy Balance

The Mechanical Energy Balance – Friction Term

(Review)

The friction has been measured and published in this form:

Straight pipes:

$$F_{\textit{straight pipes}}=\left(4f\frac{L}{D}\right)\frac{\langle v\rangle^2}{2}$$

Use literature plot of *f* as a function of Reynolds Number

Fittings and Valves:

$$F_{\substack{\textit{fittings,}\\ \textit{valves}}}=K_f\frac{\langle v\rangle^2}{2}$$

Use literature tables of K_f for laminar and turbulent flow

Energy *quantities of interest* ***Recall from CM2110 and CM3215:***

Mechanical Energy Balance

F = friction

Friction term in Mechanical Energy Balance

(see McCabe et al., or Morrison Chapter 1, or Perry's Chem Eng Handbook)

(Review)

length of straight pipe

number of each type of fitting

$$F_{friction} = \left(4f \frac{L}{D} + \sum_i K_{f_i} n_i \right) \frac{\langle v \rangle^2}{2}$$

Note f is a function of velocity)
(from literature; the Moody chart)

friction-loss coefficients
(from literature; see McCabe et al., Geankoplis, or Morrison Chapter 1)

Note that friction overall is directly a function of velocity)

If the velocity changes within the system (e.g. pipe diameter changes), then we need different friction terms for each velocity

---

Energy *quantities of interest* ***Recall from CM2110 and CM3215:***

Mechanical Energy Balance

Data are organized in terms of two dimensionless parameters:

(Review)

Flow rate

Reynolds Number

$$\mathrm{Re} = \frac{\rho \langle v_z \rangle D}{\mu}$$

Pressure Drop

Fanning Friction Factor

$$f = \frac{\frac{1}{4}(P_0 - P_L)}{\left(\frac{L}{D}\right)\left(\frac{1}{2}\rho \langle v_z \rangle^2\right)}$$

ρ – *density*
$\langle v_z \rangle$ – *average velocity*
D – *pipe diameter*
μ – *viscosity*
$P_0 - P_L$ – *pressure drop*
L – *pipe length*

Energy *quantities of interest* **Recall from CM2110 and CM3215:**

Mechanical Energy Balance

(Review)

Moody Chart: Data Correlation for Friction in Straight Pipes

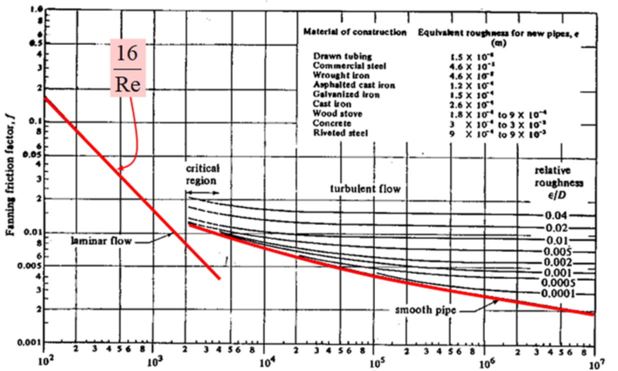


FIGURE 2.10-3. *Friction factors for fluids inside pipes. [Based on L. F. Moody, Trans. A.S.M.E., 66, 671, (1944); Mech. Eng. 69, 1005 (1947). With permission.]*

Moody Chart

(image from: Geankoplis)

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Energy *quantities of interest* **Recall from CM2110 and CM3215:**

Mechanical Energy Balance

Friction Loss from Fittings K_f

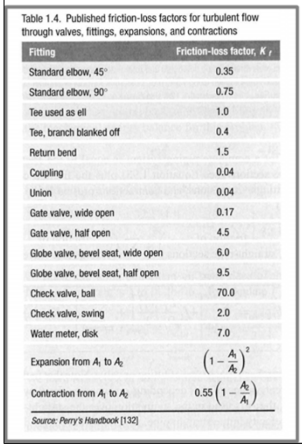

Table 1.4. Published friction-loss factors for turbulent flow through valves, fittings, expansions, and contractions

Fitting	Friction-loss factor, K_f
Standard elbow, 45°	0.35
Standard elbow, 90°	0.75
Tee used as ell	1.0
Tee, branch blanked off	0.4
Return bend	1.5
Coupling	0.04
Union	0.04
Gate valve, wide open	0.17
Gate valve, half open	4.5
Globe valve, bevel seat, wide open	6.0
Globe valve, bevel seat, half open	9.5
Check valve, ball	70.0
Check valve, swing	2.0
Water meter, disk	7.0
Expansion from A_1 to A_2	$\left(1-\frac{A_1}{A_2}\right)^2$
Contraction from A_1 to A_2	$0.55\left(1-\frac{A_2}{A_1}\right)$

Source: Perry's Handbook [132]

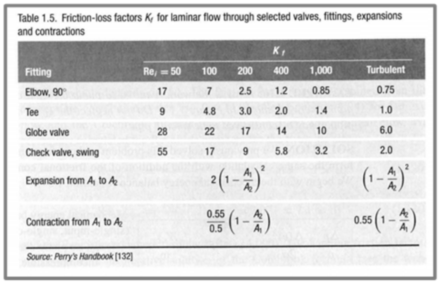

Table 1.5. Friction-loss factors K_f for laminar flow through selected valves, fittings, expansions and contractions

	K_f					
Fitting	$Re_l = 50$	100	200	400	1,000	Turbulent
Elbow, 90°	17	7	2.5	1.2	0.85	0.75
Tee	9	4.8	3.0	2.0	1.4	1.0
Globe valve	28	22	17	14	10	6.0
Check valve, swing	55	17	9	5.8	3.2	2.0
Expansion from A_1 to A_2			$2\left(1-\frac{A_1}{A_2}\right)^2$			$\left(1-\frac{A_1}{A_2}\right)^2$
Contraction from A_1 to A_2			$\frac{0.55}{0.5}\left(1-\frac{A_2}{A_1}\right)$			$0.55\left(1-\frac{A_2}{A_1}\right)$

Source: Perry's Handbook [132]

(source: Morrison, Chapter 1; originally from Perry's Handbook)

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***Energy** quantities of interest*

Recall from CM2110 and CM3215:

Open system, steady, macroscopic energy balance on mechanical systems (a.k.a.)
Mechanical Energy Balance, MEB

1. single-input, single-output
2. steady state
3. constant density (incompressible fluid)
4. temperature approximately constant
5. No phase change
6. No chemical reaction
7. insignificant amounts of heat transferred

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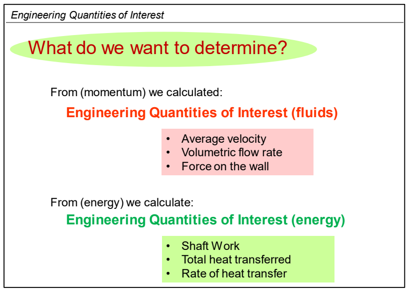
4. *temperature* approximately constant
5. *No phase change*
6. *No chemical reaction*
7. insignificant amounts of *heat transferred*

Although ChemEs care about these **mechanical systems**, we care equally (or more?) about:

- Reactors
- Separators
- Heaters
- Dryers
- Evaporators
- Coolers
- Unsteady states
- Complex inflow/outflow…

Energy *quantities of interest*

Where are we?



Engineering Quantities of Interest

What do we want to determine?

From (momentum) we calculated:

Engineering Quantities of Interest (fluids)

- Average velocity
- Volumetric flow rate
- Force on the wall

From (energy) we calculate:

Engineering Quantities of Interest (energy)

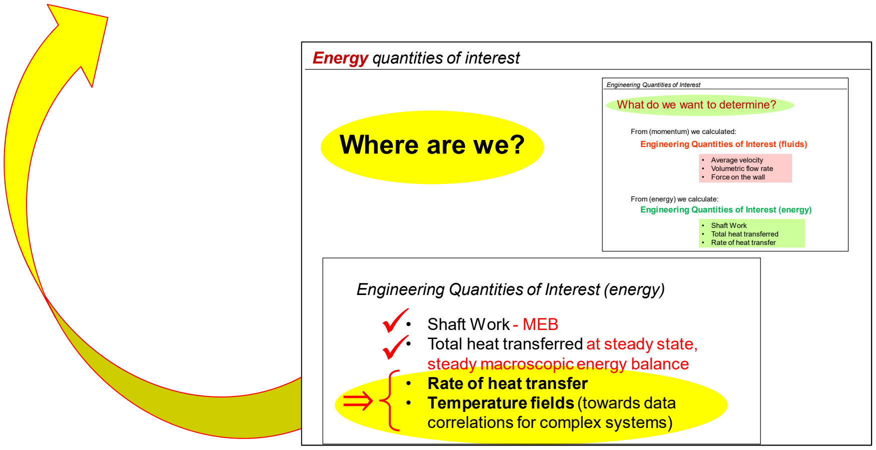
- Shaft Work
- Total heat transferred
- Rate of heat transfer

Engineering Quantities of Interest (energy)

- ✓ Shaft Work - MEB
- ✓ Total heat transferred at steady state, steady macroscopic energy balance
- ⇒ **Rate of heat transfer**
- ⇒ **Temperature fields** (towards data correlations for complex systems)

Energy *quantities of interest*

To determine **heat transfer rates** and the related **performance** of chemical engineering **unit operations**, we need knowledge of the temperature field and the characteristics of unsteady heat flows for ideal and complex engineering units.



Energy *quantities of interest*

Where are we?

Engineering Quantities of Interest

What do we want to determine?

From (momentum) we calculated:

Engineering Quantities of Interest (fluids)

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Engineering Quantities of Interest (energy)

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- ⇒ **Rate of heat transfer**
- ⇒ **Temperature fields** (towards data correlations for complex systems)

***Energy** quantities of interest*

Engineering Quantities of Interest

(energy)

Rate of heat transfer, conduction

$$\frac{q}{A} = \underset{=}{\tilde{q}} = -k\nabla T$$

Fourier's law

Total heat flow, general

$$Q = \iint_S \left[\hat{n} \cdot \underset{=}{\tilde{q}}\right]_{surface} dS$$

Total heat flow, pipe

$$Q = \int_0^{2\pi}\int_0^{L} -k \left.\frac{\partial T}{\partial r}\right|_{r=R} R dz d\theta$$

***Energy** quantities of interest*

The Plan (Heat Transfer)

1. What is the general energy balance equation we will use? Where does it come from?
2. ✓ *Apply to steady macroscopic control volumes* **(CM2110)**
3. ✓ *Apply to single-input, single-output, etc.(MEB)* **(CM2120, CM3215)**
4. Apply to microscopic control volumes
5. Transport law (Fourier's law of Heat Conduction)
6. Solve for temperature fields (steady) **(Unsteady, CM3120)**
7. Calculate engineering quantities of interest (total heat transferred; heat transfer coefficient)
8. Determine (with Dimensional Analysis) correlations for complex systems involving:
    a. Forced convection
    b. Free convection
    c. Phase change
    d. Radiation

DONE

Recall from **CM2110**

✓2. *Apply to steady macroscopic control volumes*

Closed system energy balance

$$\Delta E_p + \Delta E_k + \Delta U = Q_{in} + W_{on}$$ (final-initial)

Open system energy balance

$$\Delta E_p + \Delta E_k + \Delta H = Q_{in} + W_{s,on}$$ (out-in)

- Multiple inlets and outlets ($\Delta = \Sigma_{outs} - \Sigma_{ins}$)
- Steady state
- Constant density

Review:

www.chem.mtu.edu/~fmorriso/cm310/Energy_Balance_Notes_2008.pdf

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DONE

Recall from **CM2110 and CM3215**

✓3. *Apply to single-input, single-output, etc.(MEB)*

The Mechanical Energy Balance

$$\frac{\Delta p}{\rho} + \frac{\Delta \langle v \rangle^2}{2\alpha} + g\Delta z + F = \frac{W_{s,on}}{\dot{m}}$$

$F =$ friction

$$\frac{p_2 - p_1}{\rho} + \frac{\langle v \rangle_2^2 - \langle v \rangle_1^2}{2\alpha} + g(z_2 - z_1) + F_{21} = \frac{W_{s,on,21}}{\dot{m}}$$

Assumptions:

1. single-input, single output
2. Steady state
3. Constant density (incompressible fluid)
4. Temperature approximately constant
5. No phase change, no chemical reaction
6. Insignificant amounts of heat transferred

$F =$ friction

Friction term in Mechanical Energy Balance (see McCabe et al., or Morrison Chapter 1, or Perry's Chem Eng Handbook)

(Review)

length of straight pipe

number of each type of fitting

$$F_{friction} = \left(4f \frac{L}{D} + \sum_i K_{f_i} n_i \right) \frac{\langle v \rangle^2}{2}$$

Note *f* is a function of velocity) (from literature; the Moody chart)

friction-loss coefficients (from literature; see McCabe et al., Geankoplis, or Morrison Chapter 1)

Note that friction overall is directly a function of velocity)

If the velocity changes within the system (e.g. pipe diameter changes), then we need different friction terms for each velocity

Moody Chart: Data Correlation for Friction in Straight Pipes

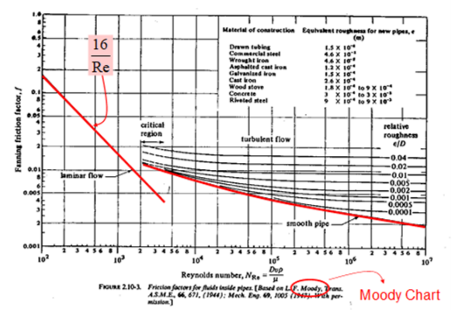


FIGURE 2.10-3. *Friction factors for fluids inside pipes. [Based on L. F. Moody, Trans. A.S.M.E., 66, 671, (1944); Mech. Eng. 69, 1005 (1947). With permission.]*

(Where did all this come from? We saw in Part 1: dimensional analysis.)

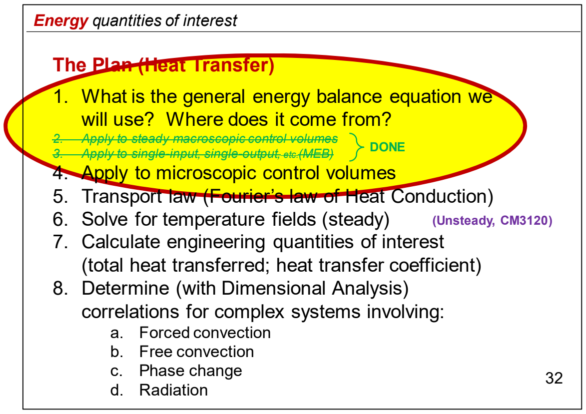
***Energy** quantities of interest*

The Plan (Heat Transfer)

1. What is the general energy balance equation we will use? Where does it come from?
~~2. Apply to steady macroscopic control volumes~~
~~3. Apply to single-input, single-output, etc.(MEB)~~ } DONE
4. Apply to microscopic control volumes
5. Transport law (Fourier's law of Heat Conduction)
6. Solve for temperature fields (steady) **(Unsteady, CM3120)**
7. Calculate engineering quantities of interest (total heat transferred; heat transfer coefficient)
8. Determine (with Dimensional Analysis) correlations for complex systems involving:
    a. Forced convection
    b. Free convection
    c. Phase change
    d. Radiation

***Energy** quantities of interest*

Let's Begin.



***Energy** quantities of interest*

The Plan (Heat Transfer)

1. What is the general energy balance equation we will use? Where does it come from?
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8. Determine (with Dimensional Analysis) correlations for complex systems involving:
    a. Forced convection
    b. Free convection
    c. Phase change
    d. Radiation

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1. and 4. What is the general energy balance equation for microscopic control volumes, and where does it come from?

What is the general energy balance equation and where does it come from?

First Law of Thermodynamics:
(**on a body**)

$$\boxed{\frac{dE_B}{dt} = Q_{in,B} - W_{by,B}}$$

First Law of Thermodynamics:
(**on a control volume**)

$$\frac{dE_{CV}}{dt} = Q_{in,CV} - W_{by,CV} + \underbrace{\iint_{CS} -(\hat{n}\cdot\underline{v})\rho\hat{E}\,dS}_{\text{the } \textit{usual} \text{ convective term: net energy convected in}}$$

Reference for derivation: Morrison, F. A., Web Appendix D1: Microscopic Energy Balance, Supplement to *An Introduction to Fluid Mechanics* (Cambridge, 2013), www.chem.mtu.edu/~fmorriso/IFM_WebAppendixD2011.pdf

---

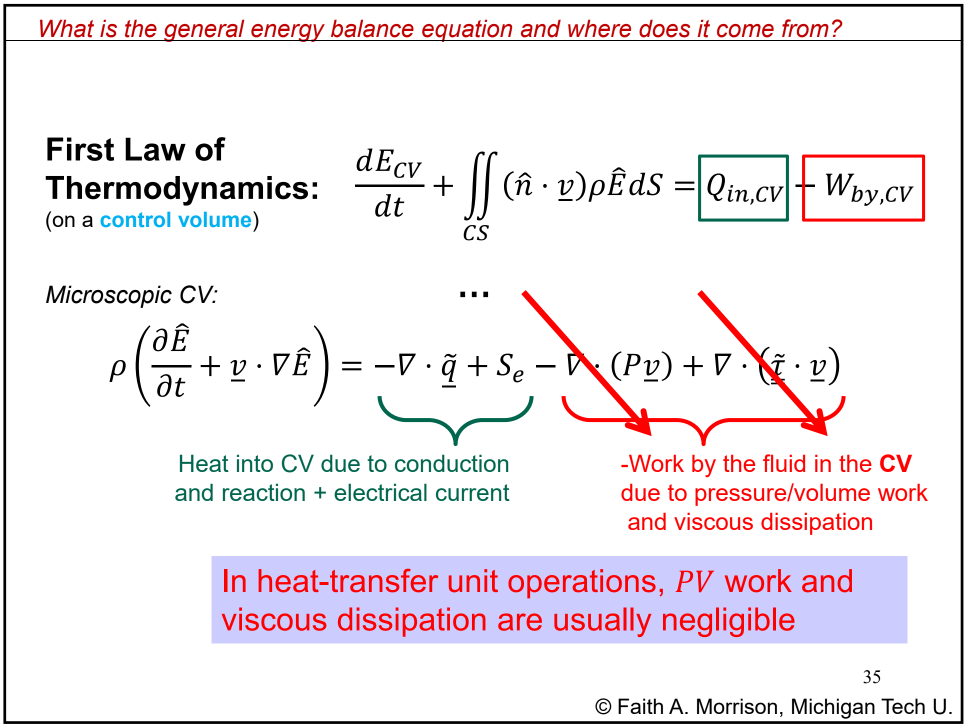
What is the general energy balance equation and where does it come from?

First Law of Thermodynamics:
(on a **control volume**)

$$\frac{dE_{CV}}{dt} + \iint_{CS} (\hat{n}\cdot\underline{v})\rho\hat{E}\,dS = \boxed{Q_{in,CV}} - \boxed{W_{by,CV}}$$

Microscopic CV: …

$$\rho\left(\frac{\partial\hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = \underbrace{-\nabla\cdot\underline{\tilde{q}} + S_e}_{\substack{\text{Heat into CV due to conduction}\\ \text{and reaction + electrical current}}} \underbrace{-\nabla\cdot(P\underline{v}) + \nabla\cdot(\underline{\underline{\tilde{\tau}}}\cdot\underline{v})}_{\substack{\text{-Work by the fluid in the CV}\\ \text{due to pressure/volume work}\\ \text{and viscous dissipation}}}$$



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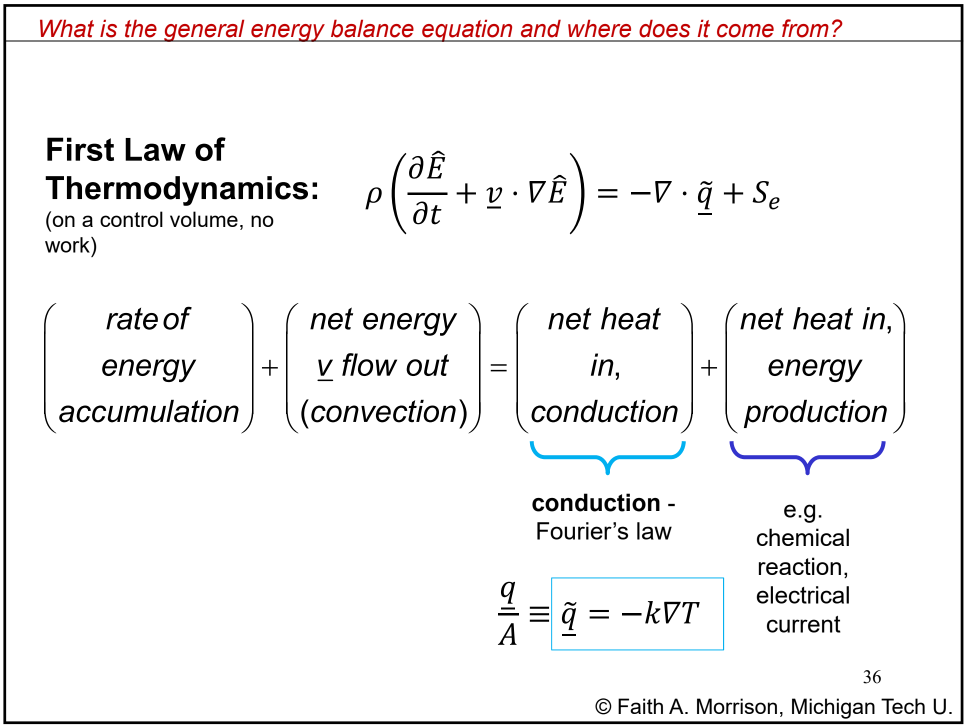
$$\frac{dE_{CV}}{dt} + \iint_{CS} (\hat{n}\cdot\underline{v})\rho\hat{E}\,dS = \boxed{Q_{in,CV}} - \boxed{W_{by,CV}}$$

Microscopic CV:

…

$$\rho\left(\frac{\partial\hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = \underbrace{-\nabla\cdot\tilde{\underline{q}} + S_e}_{\text{Heat into CV due to conduction and reaction + electrical current}} \underbrace{-\cancel{\nabla\cdot(P\underline{v})} + \cancel{\nabla\cdot(\tilde{\underline{\tau}}\cdot\underline{v})}}_{\text{-Work by the fluid in the CV due to pressure/volume work and viscous dissipation}}$$

In heat-transfer unit operations, *PV* work and viscous dissipation are usually negligible



What is the general energy balance equation and where does it come from?

First Law of Thermodynamics:
(on a control volume, no work)

$$\rho\left(\frac{\partial\hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\tilde{\underline{q}} + S_e$$

$$\begin{pmatrix}\text{rate of}\\ \text{energy}\\ \text{accumulation}\end{pmatrix} + \begin{pmatrix}\text{net energy}\\ \underline{v}\text{ flow out}\\ \text{(convection)}\end{pmatrix} = \underbrace{\begin{pmatrix}\text{net heat}\\ \text{in,}\\ \text{conduction}\end{pmatrix}}_{\textbf{conduction}\text{ - Fourier's law}} + \underbrace{\begin{pmatrix}\text{net heat in,}\\ \text{energy}\\ \text{production}\end{pmatrix}}_{\text{e.g. chemical reaction, electrical current}}$$

$$\frac{q}{A} \equiv \boxed{\tilde{\underline{q}} = -k\nabla T}$$

What is the general energy balance equation and where does it come from?

First Law of Thermodynamics:
(on a control volume, no work)

$$\rho\left(\frac{\partial \hat{E}}{\partial t} + \underline{v}\cdot\nabla\hat{E}\right) = -\nabla\cdot\underline{\tilde{q}} + S_e$$

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conduction - Fourier's law

e.g. chemical reaction, electrical current

Note the two different q's (watch units)

$$\frac{\underline{q}}{A} \equiv \boxed{\underline{\tilde{q}} = -k\nabla T}$$

What is the general energy balance equation and where does it come from?

First Law of Thermodynamics:
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conduction - Fourier's law

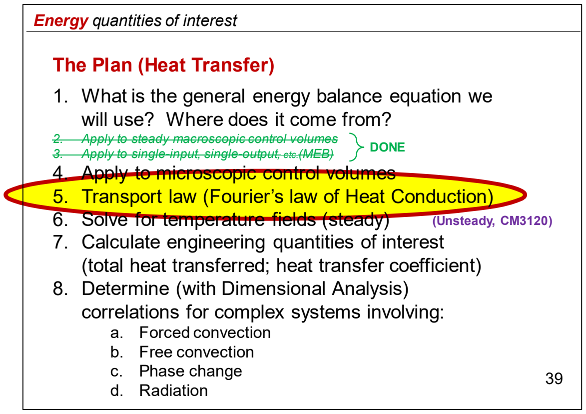
e.g. chemical reaction, electrical current

Other energy contributions will enter through the ***boundary conditions****.*

$$\frac{\underline{q}}{A} \equiv \boxed{\underline{\tilde{q}} = -k\nabla T}$$

***Energy** quantities of interest*

Continuing…



***Energy** quantities of interest*

The Plan (Heat Transfer)

1. What is the general energy balance equation we will use? Where does it come from?
2. ~~Apply to steady macroscopic control volumes~~
3. ~~Apply to single-input, single-output, etc.(MEB)~~

DONE

4. Apply to microscopic control volumes
5. Transport law (Fourier's law of Heat Conduction)
6. Solve for temperature fields (steady) **(Unsteady, CM3120)**
7. Calculate engineering quantities of interest (total heat transferred; heat transfer coefficient)
8. Determine (with Dimensional Analysis) correlations for complex systems involving:
   - a. Forced convection
   - b. Free convection
   - c. Phase change
   - d. Radiation

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5. What is the energy transport law?

Transport law (Fourier's law of Heat Conduction)

Part I: Momentum Transfer

Momentum transfer:

viscosity

$$\tau_{21} = \underbrace{(-\tilde{\tau}_{21})}_{\text{momentum flux}} = -\mu\underbrace{\left(\frac{dv_1}{dx_2}\right)}_{\text{velocity gradient}}$$

Part II: Heat Transfer

Heat transfer:

thermal conductivity

$$\underbrace{\frac{q_x}{A}}_{\text{heat flux}} = -k\underbrace{\frac{dT}{dx}}_{\text{temperature gradient}}$$

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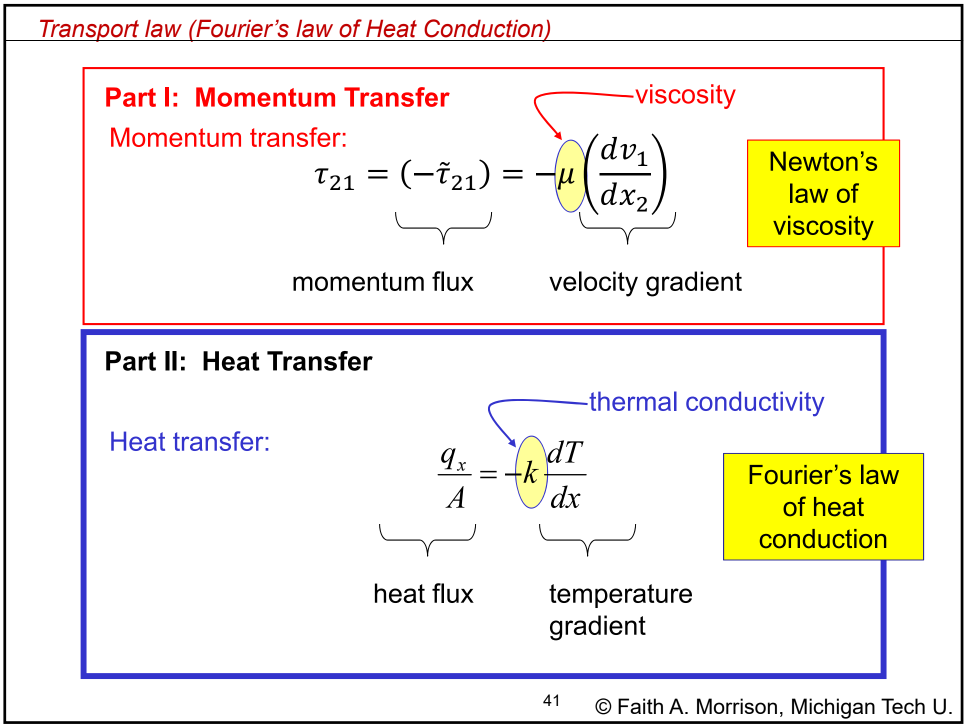

Transport law (Fourier's law of Heat Conduction)

Part I: Momentum Transfer

Momentum transfer:

$$\tau_{21} = \underbrace{(-\tilde{\tau}_{21})}_{\text{momentum flux}} = -\mu \underbrace{\left(\frac{dv_1}{dx_2}\right)}_{\text{velocity gradient}}$$

viscosity

Newton's law of viscosity

Part II: Heat Transfer

Heat transfer:

$$\underbrace{\frac{q_x}{A}}_{\text{heat flux}} = -k \underbrace{\frac{dT}{dx}}_{\text{temperature gradient}}$$

thermal conductivity

Fourier's law of heat conduction

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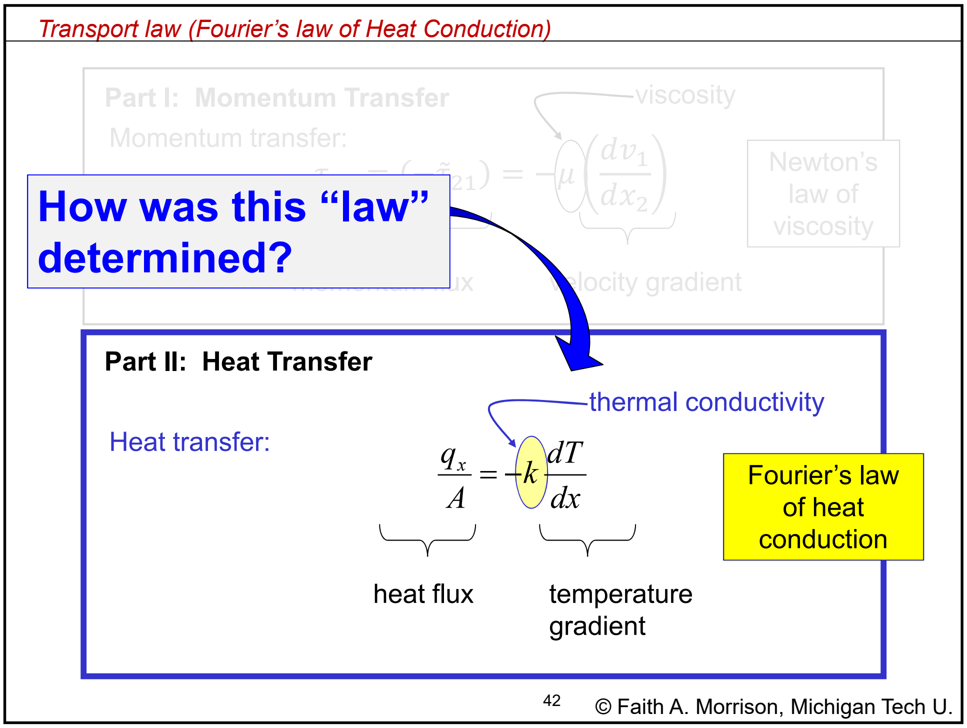

Transport law (Fourier's law of Heat Conduction)

Part I: Momentum Transfer

Momentum transfer:

$$\left(-\tilde{\tau}_{21}\right) = -\mu \left(\frac{dv_1}{dx_2}\right)$$

viscosity

Newton's law of viscosity

ux

locity gradient

How was this "law" determined?

Part II: Heat Transfer

Heat transfer:

$$\underbrace{\frac{q_x}{A}}_{\text{heat flux}} = -k \underbrace{\frac{dT}{dx}}_{\text{temperature gradient}}$$

thermal conductivity

Fourier's law of heat conduction

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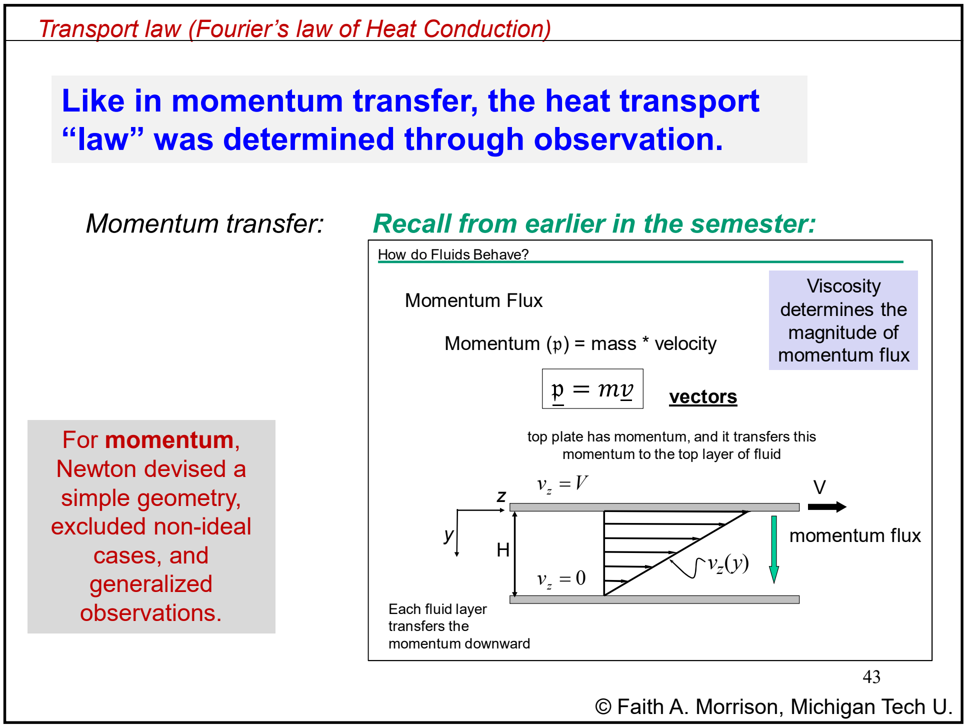

Transport law (Fourier's law of Heat Conduction)

Like in momentum transfer, the heat transport "law" was determined through observation.

Momentum transfer:

Recall from earlier in the semester:

How do Fluids Behave?

Momentum Flux

Momentum (p) = mass * velocity

$\underline{p} = m\underline{v}$ **vectors**

Viscosity determines the magnitude of momentum flux

top plate has momentum, and it transfers this momentum to the top layer of fluid

Each fluid layer transfers the momentum downward

For **momentum**, Newton devised a simple geometry, excluded non-ideal cases, and generalized observations.

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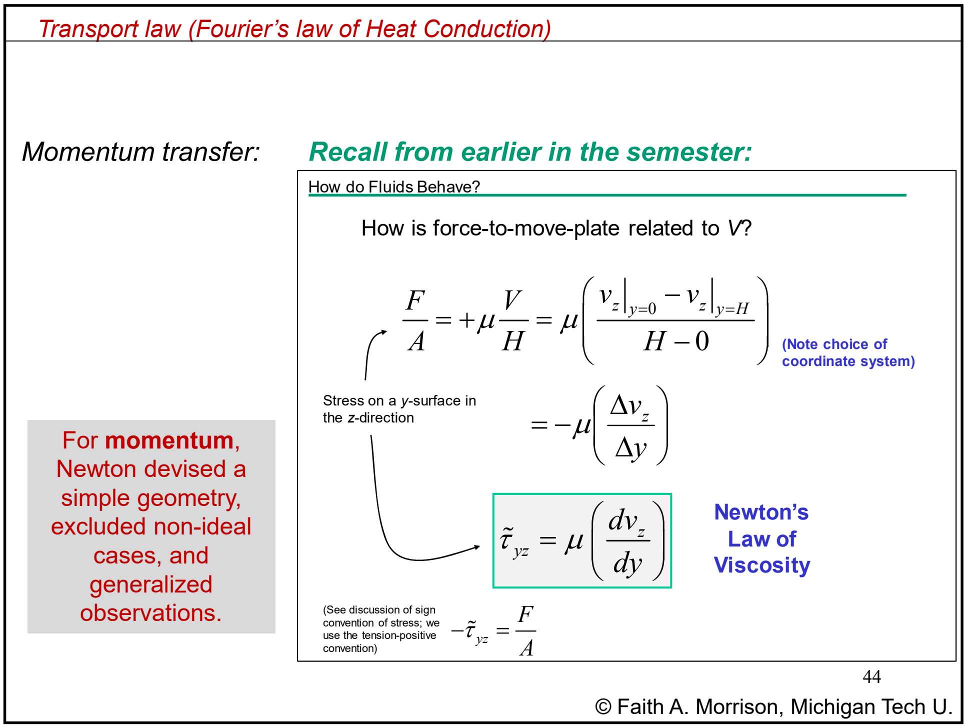

Transport law (Fourier's law of Heat Conduction)

Momentum transfer:

Recall from earlier in the semester:

How do Fluids Behave?

How is force-to-move-plate related to *V*?

$$\frac{F}{A} = +\mu\frac{V}{H} = \mu\left(\frac{v_z|_{y=0} - v_z|_{y=H}}{H-0}\right)$$

(Note choice of coordinate system)

Stress on a *y*-surface in the *z*-direction

$$= -\mu\left(\frac{\Delta v_z}{\Delta y}\right)$$

$$\boxed{\tilde{\tau}_{yz} = \mu\left(\frac{dv_z}{dy}\right)}$$

Newton's Law of Viscosity

(See discussion of sign convention of stress; we use the tension-positive convention) $-\tilde{\tau}_{yz} = \frac{F}{A}$

For **momentum**, Newton devised a simple geometry, excluded non-ideal cases, and generalized observations.

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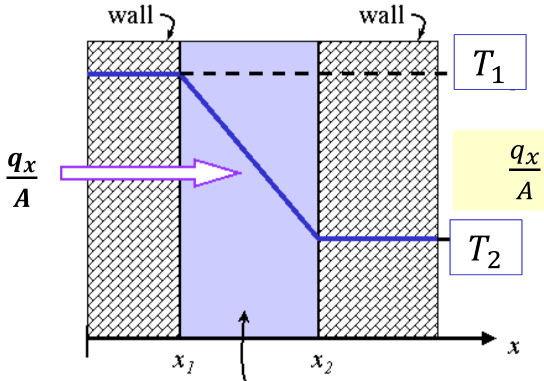
Transport law (Fourier's law of Heat Conduction)

Fourier's Experiments: Simple One-dimensional Heat Conduction

For **heat conduction**, Fourier devised a simple geometry, excluded non-ideal cases, and generalized observations.



$$\frac{q_x}{A} = k\frac{T_1 - T_2}{x_2 - x_1} = -k\frac{dT}{dx}$$

Fourier's law of heat conduction

Homogeneous material of thermal conductivity, k

Transport law (Fourier's law of Heat Conduction)

Fourier's law of Heat Conduction:

makes reference to a coordinate system

$$\frac{q_x}{A} = -k\frac{dT}{dx}$$

Allows you to solve for temperature profiles

(temperature profile = temperature distributions, temperature field, $T(x, y, z, t)$)

Gibbs notation: $\frac{q}{A} = -k\nabla T$

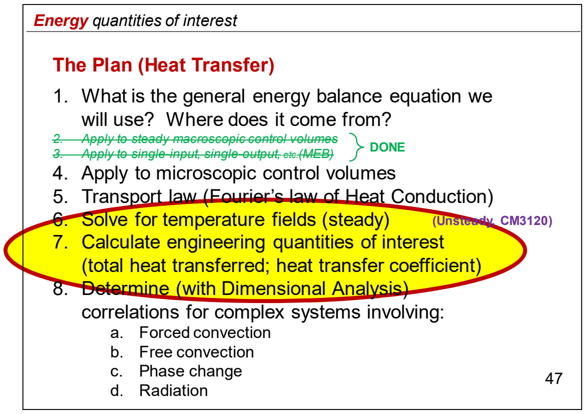
Fourier's law

$$\underset{\sim}{\tilde{q}} = \frac{\underset{\sim}{q}}{A} = \begin{pmatrix} -k\frac{\partial T}{\partial x} \\ -k\frac{\partial T}{\partial y} \\ -k\frac{\partial T}{\partial z} \end{pmatrix}_{xyz}$$

- •Heat flows **down** a temperature gradient
- •Flux is proportional to magnitude of temperature gradient

***Energy** quantities of interest*

Continuing…



Energy quantities of interest

The Plan (Heat Transfer)

1. What is the general energy balance equation we will use? Where does it come from?
~~2. Apply to steady macroscopic control volumes~~
~~3. Apply to single-input, single-output, etc.(MEB)~~ } DONE
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    c. Phase change
    d. Radiation

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6. and 7. How do we solve for $T(x, y, z)$?
What boundary conditions do we use?

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Solve for the Temperature Field

As was true in momentum transfer (fluid mechanics) solving problems with shell balances on individual microscopic control volumes is tedious, and it is easy to make errors.

Instead, we use the general equation, derived for all control volumes:

General Energy Transport Equation
(microscopic energy balance)

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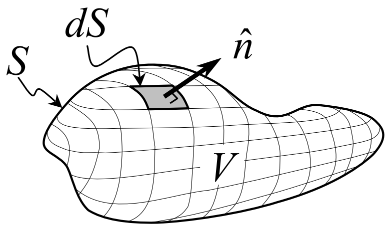
Recall Microscopic Momentum Balance:

Equation of Motion



Microscopic **momentum** balance written on an arbitrarily shaped control volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\nabla\cdot\underline{\underline{\tilde{\tau}}}+\rho\underline{g}$$ **general fluid**

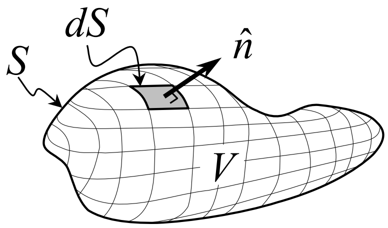
Gibbs notation: $$\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\mu\nabla^{2}\underline{v}+\rho\underline{g}$$ **Newtonian fluid**

Navier-Stokes Equation

Microscopic momentum balance is a vector equation.

Microscopic Energy Balance:

Equation of Thermal Energy



Microscopic **energy** balance written on an arbitrarily shaped volume, *V*, enclosed by a surface, *S*

Gibbs notation: $$\rho\left(\frac{\partial \hat{E}}{\partial t}+\underline{v}\cdot\nabla\hat{E}\right)=-\nabla\cdot\underline{\tilde{q}}+S_e$$ **general conduction**

Gibbs notation: $$\rho\hat{C}_p\left(\frac{\partial T}{\partial t}+\underline{v}\cdot\nabla T\right)=k\nabla^{2}T+S_e$$ **Only Fourier conduction**

(incompressible fluid, constant pressure, neglect $\hat{E}_k$, $\hat{E}_p$, viscous dissipation)

Solve for the Temperature Field

Equation of Energy

(microscopic energy balance)

$$\rho \hat{C}_p \left(\underbrace{\frac{\partial T}{\partial t}}_{\text{rate of change}} + \overbrace{\underline{v} \cdot \nabla T}^{\text{convection}} \right) = \underbrace{k \nabla^2 T}_{\text{conduction (all directions)}} + \overbrace{S_e}^{\text{source}}$$

(energy generated per unit volume per time)

velocity must satisfy equation of motion, equation of continuity

see handout for component notation

Solve for the Temperature Field

How do we solve for temperature fields?

How did we solve for velocity and stress fields?

Recall from earlier in the semester:

Problem-Solving Procedure – *solving for velocity and stress fields*

1. sketch system
2. choose coordinate system
3. choose a control volume
4. perform a mass balance
5. perform a momentum balance
   (will contain stress)
6. substitute in *Newton's law of viscosity*, e.g. $\boxed{\tilde{\tau}_{xz} = \mu \left(\frac{dv_z}{dx} \right)}$
7. solve the <u>differential</u> equation
8. apply boundary conditions

Solve for the Temperature Field

How did we solve for ~~velocity and stress~~ temperature fields?

Problem-Solving Procedure – *solving ~~for velocity and stress~~ temperature fields*

1. sketch system
2. choose coordinate system
3. choose a control volume
4. perform a mass balance
5. perform ~~a momentum~~ energy balance
   (will contain ~~stress)~~ heat flux
6. substitute in ~~*Newton's law of viscosity,* e.g. $\tilde{\tau}_{xz} = -\mu\left(\frac{dv_z}{dx}\right)$~~
7. solve the <u>differential</u> equation
8. apply boundary conditions

Fourier's law of heat conduction, $\tilde{q} = -k\frac{dT}{dx}$

Solve for the Temperature Field

Problem-Solving Procedure – *Solving for Temperature Fields*

1. sketch system
2. choose coordinate system
3. choose a control volume
4. perform a mass balance
5. perform an energy balance
   (will contain heat flux)
6. substitute in *Fourier's law of heat conduction,* e.g. $\tilde{q}_x = -k\frac{dT}{dx}$
7. solve the <u>differential</u> equation
8. apply boundary conditions

Microscopic Energy Balance

The **Equation of Energy** for systems with **constant k**

Microscopic energy balance, constant thermal conductivity; Gibbs notation

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + \underline{v} \cdot \nabla T \right) = k \nabla^2 T + S$$

Microscopic energy balance, constant thermal conductivity; Cartesian coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; cylindrical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right) + S$$

Microscopic energy balance, constant thermal conductivity; spherical coordinates

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin\theta} \frac{\partial T}{\partial \phi} \right)$$
$$= k \left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 T}{\partial \phi^2} \right) + S$$

Note: this handout is also on the web

https://pages.mtu.edu/~fmorriso/cm310/energy.pdf

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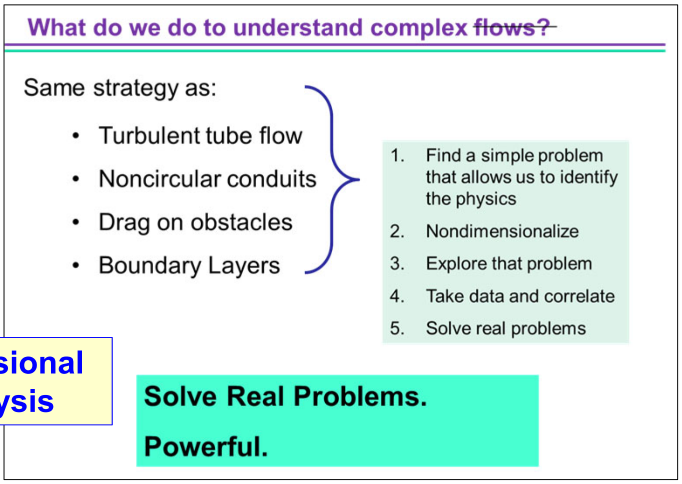
Solve for the Temperature Field

For more complex problems:

Recall from earlier in the semester: heat transfer



What do we do to understand complex ~~flows?~~

Same strategy as:

- Turbulent tube flow
- Noncircular conduits
- Drag on obstacles
- Boundary Layers

1. Find a simple problem that allows us to identify the physics
2. Nondimensionalize
3. Explore that problem
4. Take data and correlate
5. Solve real problems

Dimensional Analysis

Solve Real Problems.

Powerful.

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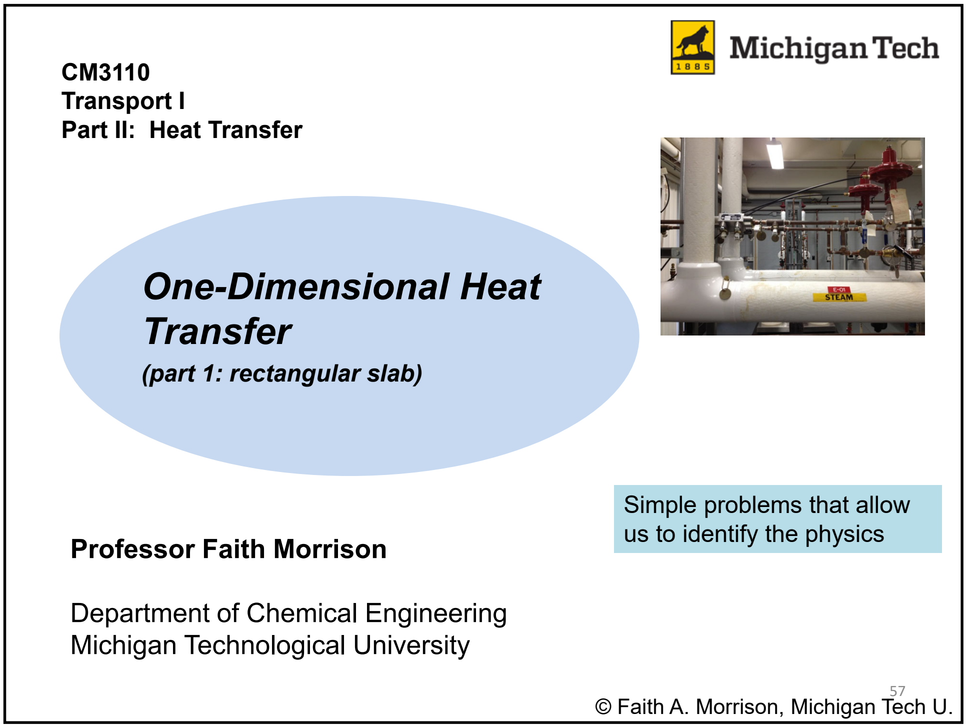


CM3110
Transport I
Part II: Heat Transfer

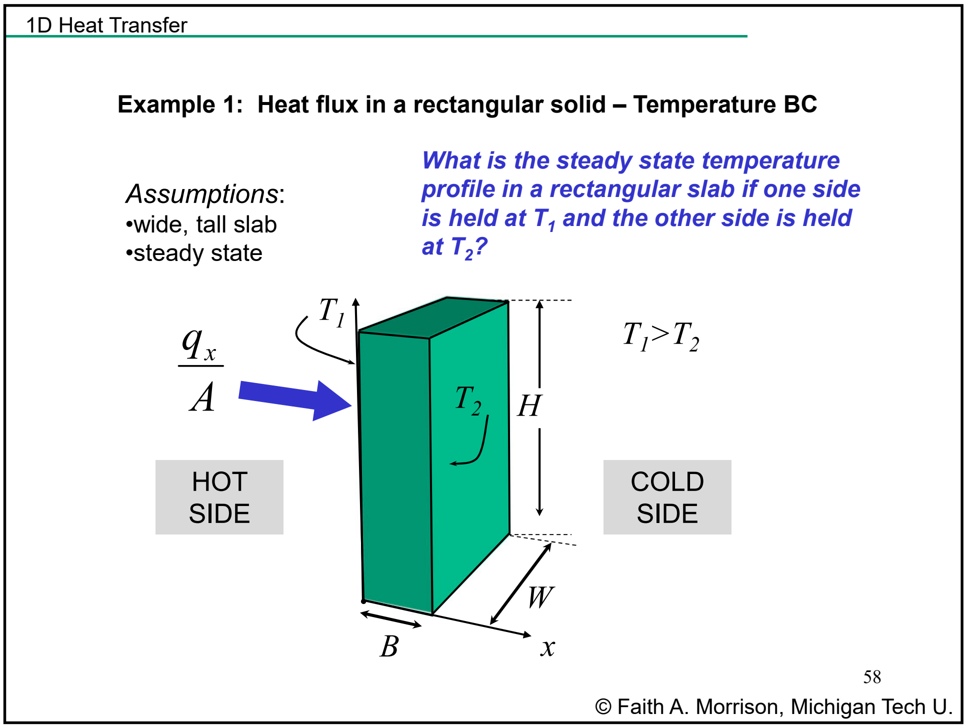
One-Dimensional Heat Transfer

(part 1: rectangular slab)

Simple problems that allow us to identify the physics

Professor Faith Morrison

Department of Chemical Engineering
Michigan Technological University



1D Heat Transfer

Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
- wide, tall slab
- steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

1D Heat Transfer

Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
- wide, tall slab
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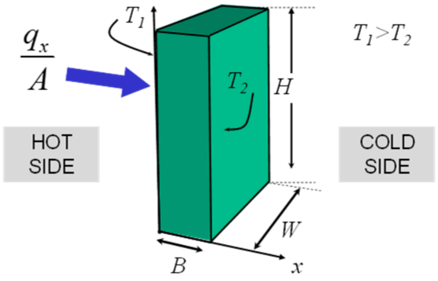
What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?



Let's try.

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

1D Heat Transfer

Example 1: Heat flux in a rectangular solid – Temp BC

Result:

$$T = \tilde{c}_1 x + \tilde{c}_2$$

$$\frac{q_x}{A} = -k\left(\frac{dT}{dx}\right) = -k\tilde{c}_1$$

Fourier's law

(starting with the temperature version of the micro E bal)

Constant

Boundary conditions?

Note: different integration constants are defined when we use the ***temperature*** *version and the* ***flux*** *version of the microscopic energy balance; after boundary conditions are applied, the answer is the same.*

1D Heat Transfer

Example 1: Heat flux in a rectangular solid – Temp BC

Solution:

$$\frac{q_x}{A} = -k\left(\frac{T_2 - T_1}{B}\right)$$

$$T = \left(\frac{T_2 - T_1}{B}\right)x + T_1$$

Flux is constant, and depends on *k*

Temp. profile varies linearly, and **does not** depend on *k*

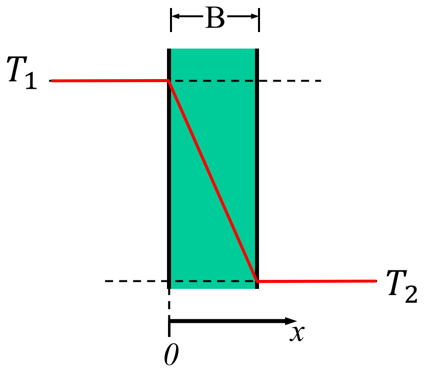
1D Heat Transfer

Example 1: Heat flux in a rectangular solid – Temp BC

<u>SOLUTION</u>:

$$T = \frac{(T_2 - T_1)}{B}x + T_1$$

$$\frac{q_x}{A} = -k\frac{(T_2 - T_1)}{B}$$

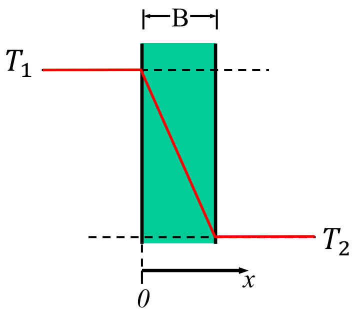


1D Heat Transfer

Example 1: Heat flux in a slab

Using the solution (conceptual):

For heat conduction in a slab with temperature boundary conditions, we sketched the solution as shown. **If the thermal conductivity *k* of the slab became larger**, how would the sketch change? What are the predictions for *T(x)* and the flux for this case?

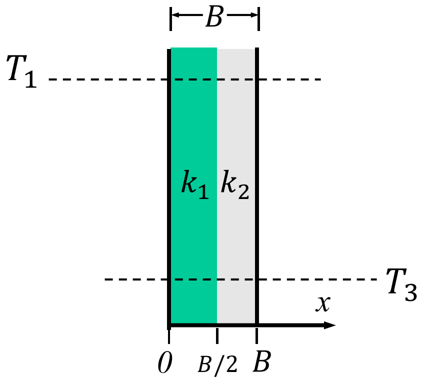


Let's try.

Using the solution: Composite Door:

For an outside door, a metal is used (k_1) for strength, and a cork k_2) is used for insulation. Both are the same thickness $B/2$. What is the temperature profile in the door at steady state? What is the flux? The inside temperature of the metal is T_1 and the outside temperature of the cork is T_3.

$k_1 \gg k_2$



Let's try.

Note: *in the hand notes the temperatures from left to right are* T_1, T_3, T_2.

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

1D Heat Transfer

Example 1b: Composite Door (two equal width layers)

SOLUTION:

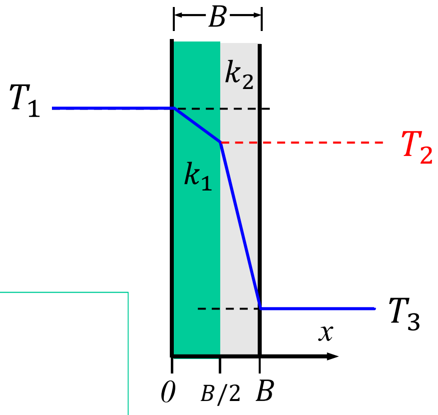
$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\left(\frac{\frac{B}{2}(k_1 + k_2)}{k_1 k_2}\right)}$$

k_1 material: $(0 \leq x \leq B/2)$

$$T(x) = \frac{(T_2 - T_1)}{B/2}x + T_1$$

k_2 material: $(B/2 \leq x \leq B)$

$$T(x) = \frac{(T_3 - T_2)}{B/2}x + (2T_2 - T_3)$$



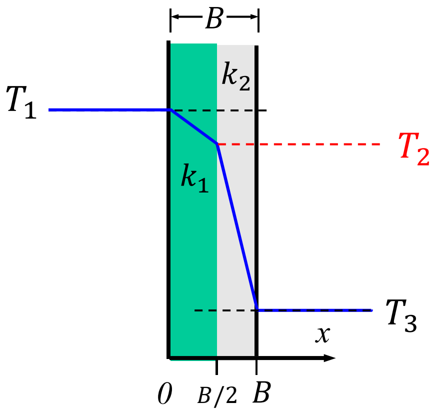
$$T_2 = \frac{k_1 T_1 + k_2 T_3}{k_1 + k_2}$$

1D Heat Transfer

Example 1b: Composite Door (two equal width layers)

SOLUTION:

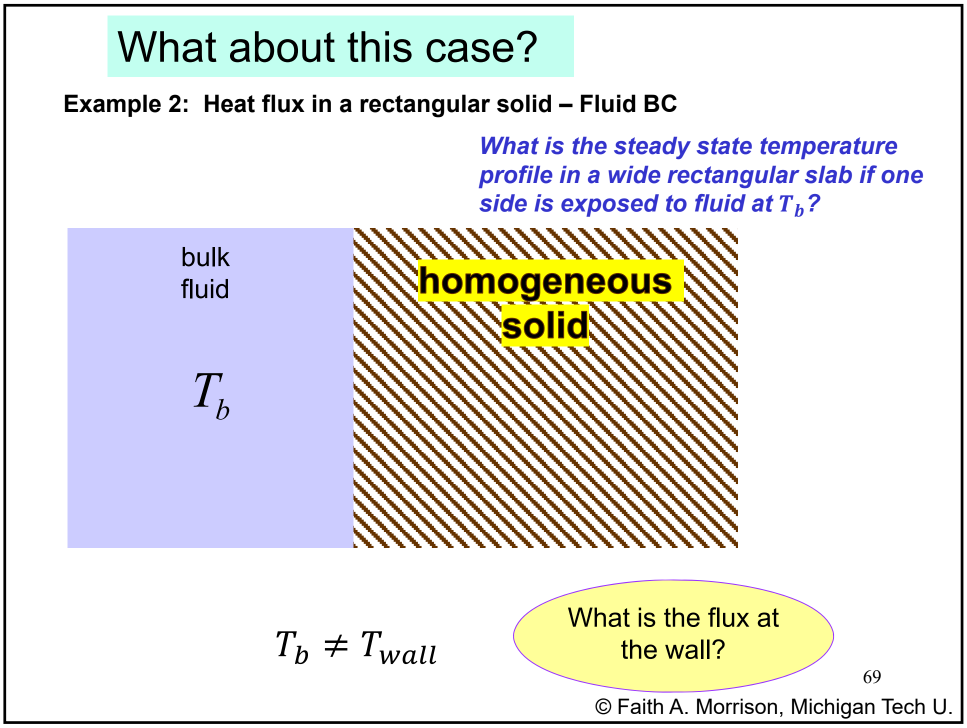
$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\frac{B/2}{k_1} + \frac{B/2}{k_2}}$$



Let: $\mathcal{R}_i \equiv \frac{\Delta x}{k_i}$

$$\frac{q_x}{A} = \frac{(T_1 - T_3)}{\mathcal{R}_1 + \mathcal{R}_2} = \frac{\text{driving force}}{\text{resistance}}$$

Each of the layers contributes a resistance, added in *series* (like in electricity).



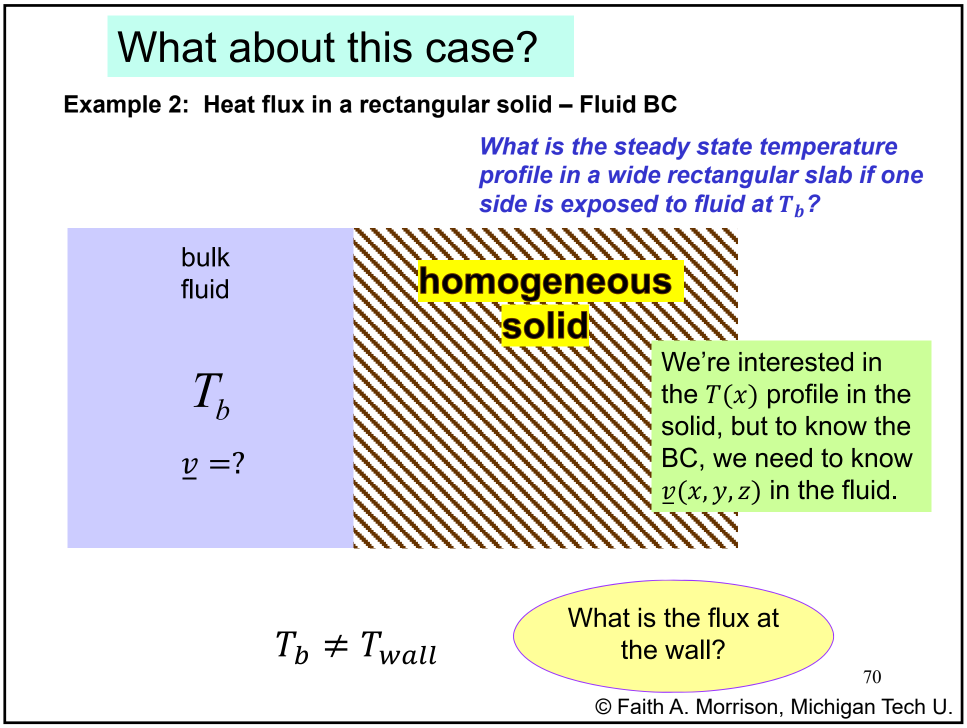
What about this case?

Example 2: Heat flux in a rectangular solid – Fluid BC

What is the steady state temperature profile in a wide rectangular slab if one side is exposed to fluid at T_b?

$T_b \neq T_{wall}$

What is the flux at the wall?



What about this case?

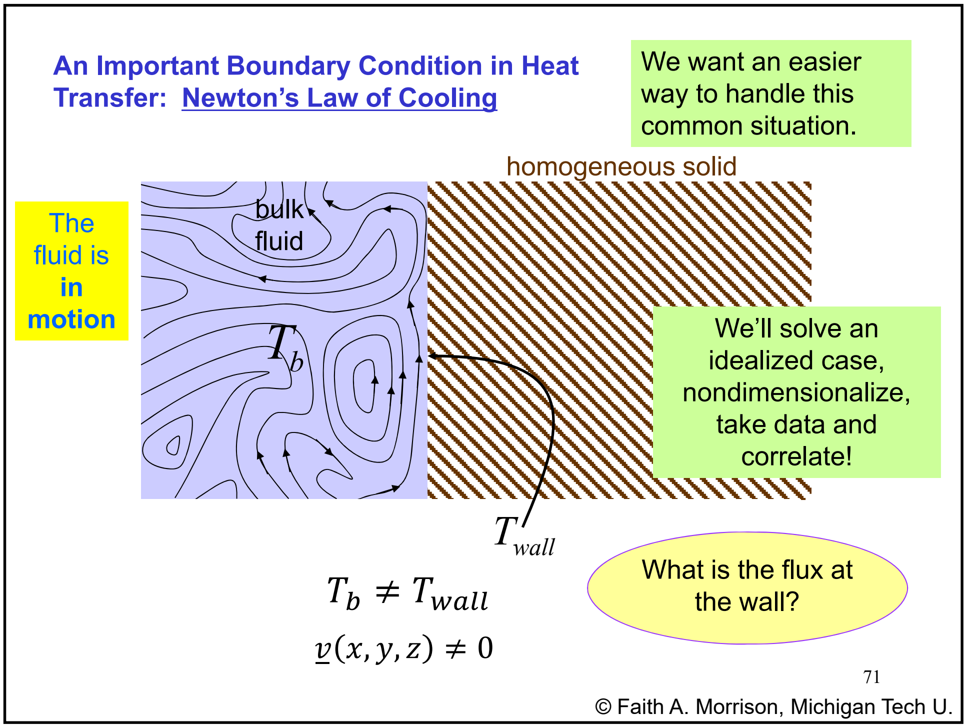
Example 2: Heat flux in a rectangular solid – Fluid BC

What is the steady state temperature profile in a wide rectangular slab if one side is exposed to fluid at T_b?

We're interested in the $T(x)$ profile in the solid, but to know the BC, we need to know $\underline{v}(x, y, z)$ in the fluid.

$T_b \neq T_{wall}$

What is the flux at the wall?



An Important Boundary Condition in Heat Transfer: Newton's Law of Cooling

We want an easier way to handle this common situation.

The fluid is **in motion**

homogeneous solid

bulk fluid

T_b

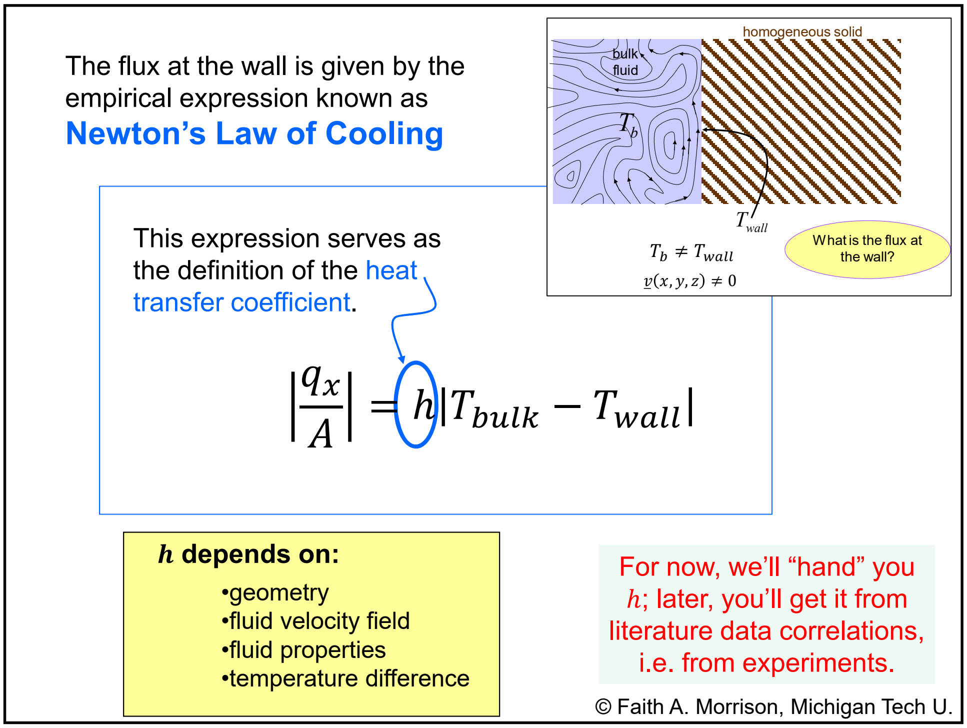
T_{wall}

We'll solve an idealized case, nondimensionalize, take data and correlate!

$$T_b \neq T_{wall}$$

$$\underline{v}(x,y,z) \neq 0$$

What is the flux at the wall?



The flux at the wall is given by the empirical expression known as

Newton's Law of Cooling

homogeneous solid

bulk fluid

T_b

T_{wall}

$T_b \neq T_{wall}$

$\underline{v}(x,y,z) \neq 0$

What is the flux at the wall?

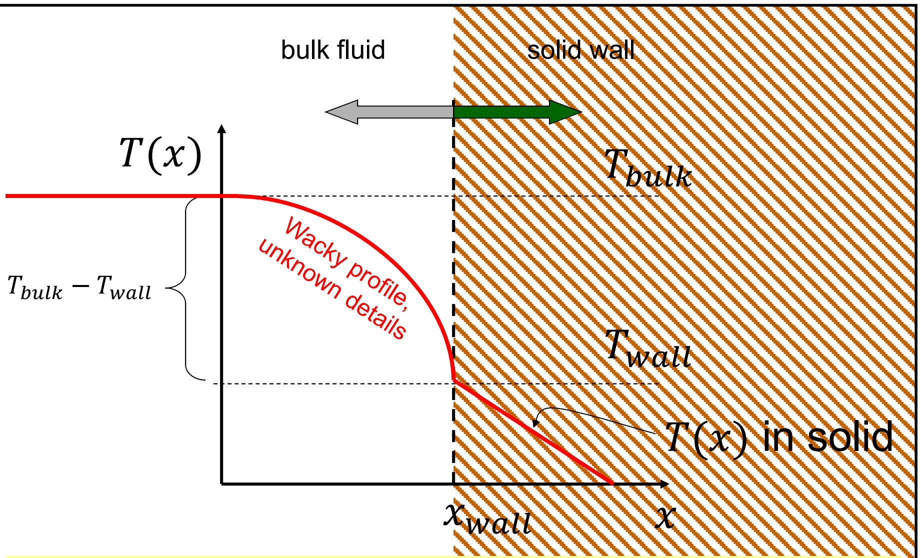
This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**

- geometry
- fluid velocity field
- fluid properties
- temperature difference

For now, we'll "hand" you h; later, you'll get it from literature data correlations, i.e. from experiments.



The temperature difference at the fluid-wall interface is caused by complex phenomena that are lumped together into the heat transfer coefficient, *h*

1D Heat Transfer

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**

$$\left|\frac{q_x}{A}\right| = h\,|T_{bulk} - T_{wall}|$$

Table 4.1-2, Approximate Magnitude of Some Heat-Transfer Coefficients

Mechanism	$h, \frac{BTU}{hr\ ft^2\ {}^oF}$	$h, \frac{W}{m^2 K}$
Condensing steam	1000-5000	5700-28,000
Condensing organics	200-500	1100-2800
Boiling liquids	300-5000	1700-28,000
Moving water	50-3000	280-17,000
Moving hydrocarbons	10-300	55-1700
Still air	0.5-4	2.8-23
Moving air	2-10	11.3-55

Reference: C. J. Geankoplis, 4th edition, Transport Processes and Separation Process Principles (includes Unit Operations), Prentice Hall, Upper Saddle River, NJ. page 241

For now, we'll "hand" you h; later, you'll get it from literature data correlations, i.e. from experiments.

1D Heat Transfer

How do we handle the <u>absolute value</u> signs?

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

- •Heat flows from hot to cold
- •Choice of direction of the coordinate system determines if the flux is positive or negative
- •Flow in the direction of increasing coordinate value means positive flux

---

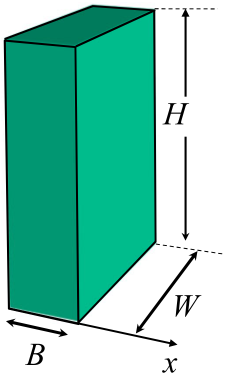
1D Heat Transfer

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Assumptions:
- •wide, tall slab
- •steady state
- •*h_1 and h_2 are the heat transfer coefficients of the left and right walls*

What is the steady state temperature profile in a rectangular slab if the <u>fluid on one side</u> is held at T_{b1} and the <u>fluid on the other side</u> is held at T_{b2}?

Bulk fluid temperature on left T_{b1}



T_{b2} Bulk fluid temperature on right

$T_{b1} > T_{b2}$

Newton's law of cooling boundary conditions

Problem-Solving Procedure – microscopic heat-transfer problems

1. sketch system
2. choose coordinate system
3. Apply the microscopic energy balance
4. solve the differential equation for temperature profile
5. apply boundary conditions
6. Calculate the flux from Fourier's law

$$\boxed{\frac{q_x}{A} = -k\frac{dT}{dx}}$$

See handwritten notes.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

1D Heat Transfer

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

(starting with the temperature version of the micro E bal)

Result:

$$T = c_1 x + c_2$$

$$\frac{q_x}{A} = -k\left(\frac{dT}{dx}\right) = -kc_1$$

Constant

Fourier's law

Boundary conditions?

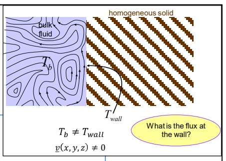
Note: different integration constants c_1, c_2 are defined when we use the temperature version of the microscopic energy balance; after boundary conditions are applied, the answer is the same.

1D Heat Transfer

Boundary conditions?

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

The flux at the wall is given by the empirical expression known as **Newton's Law of Cooling**



This expression serves as the definition of the heat transfer coefficient.

$$\left|\frac{q_x}{A}\right| = h\left|T_{bulk} - T_{wall}\right|$$

***h* depends on:**
- geometry
- fluid velocity field
- fluid properties
- temperature difference

For now, we'll "hand" you h; later, you'll get it from literature data correlations, i.e. from experiments.

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

This energy balance solution is the same as Example 1, EXCEPT there are different boundary conditions.

With Newton's law of cooling boundary condition, we know the flux at the boundary in terms of the heat transfer coefficients, h_i:

The flux is **positive** (heat flows in the $+x$-direction)

$$\left.\frac{q_x}{A}\right|_{x=0} = h_1\left(T_{b1} - T_{w1}\right) > 0$$

$$\left.\frac{q_x}{A}\right|_{x=B} = h_2\left(T_{w2} - T_{b2}\right) > 0$$

but, we do not know these temps

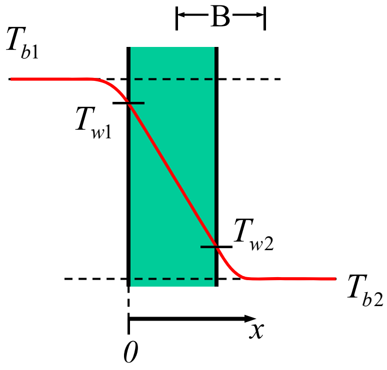
How do we apply these boundary conditions?

Soln from Example 1:

$$T = c_1 x + c_2$$

$$\frac{q_x}{A} = -k c_1$$

2 unknown constants to solve for: c_1, c_2.



We can eliminate the wall temps from the two equations for the BC by using the solution for *T(x)*.

Then solve for c_1, c_2. (2 eqns, 2 unknowns)

See handwritten notes (in class, also on web).

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

https://pages.mtu.edu/~fmorriso/cm310/algebra_details_N_law_cooling.pdf

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

After some algebra,

$$c_1 = \frac{\frac{1}{k}(T_{b2} - T_{b1})}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

$$c_2 = T_{b1} + \frac{\frac{1}{h_1}(T_{b2} - T_{b1})}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}$$

Substituting back into the solution, we obtain the final result.

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution: **(temp profile, flux)**

Temperature profile:

(linear)

$$\frac{T_{b1}-T}{T_{b1}-T_{b2}}=\frac{\frac{x}{k}+\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}$$

Flux:

(constant)

$$\frac{q_x}{A}=\frac{T_{b1}-T_{b2}}{\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}}$$

Rectangular slab with Newton's law of cooling BCs

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution: **(temp profile, flux)**

Temperature profile:

(linear)

$$\frac{T_{b1}-T}{T_{b1}-T_{b2}}=\frac{\frac{x}{k}+\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}$$

$$T=T_{b1}-\left(\frac{(T_{b1}-T_{b2})\frac{1}{k}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)x+\left(\frac{(T_{b1}-T_{b2})\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)$$

Linear temperature profile

Rectangular slab with Newton's law of cooling BCs

Example 2: Heat flux in a rectangular solid – Newton's law of cooling BC

Solution: (temp profile, flux)

Temperature profile:

(linear)

$$\frac{T_{b1}-T}{T_{b1}-T_{b2}}=\frac{\frac{x}{k}+\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}$$

$$T=T_{b1}-\left(\frac{(T_{b_1}-T_{b_2})\frac{1}{k}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)x+\left(\frac{(T_{b_1}-T_{b_2})\frac{1}{h_1}}{\left(\frac{1}{h_1}+\frac{B}{k}+\frac{1}{h_2}\right)}\right)$$

Resistance due to heat transfer coefficients

Resistance due to finite thermal conductivity

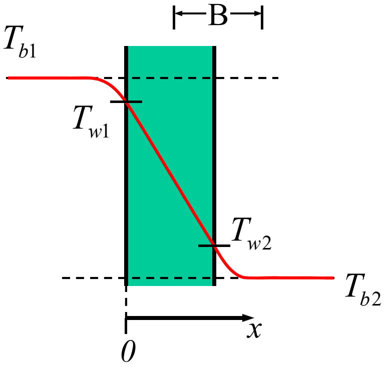
Using the solution (with numbers):

What is the temperature in the middle of a slab (thickness = B, thermal conductivity= $26\ BTU/\ h\ ft^oF$ if the left side is exposed to a fluid of temperature 120^oF and the right side is exposed to a fluid of temperature 50^oF? The heat transfer coefficients at the two faces are the same and are equal to $2.0\ BTU/h\ ft^2\ {}^oF$.

Example 4: Heat flux in a slab

Using the solution (conceptual):

For heat conduction in a slab with Newton's law of cooling boundary conditions, we sketched the solution as shown. **If the heat transfer coefficients became infinitely large**, (no change in bulk temperatures) how would the sketch change? What are the predictions for ***T(x)*** and the **flux** for this case?



Let's try.

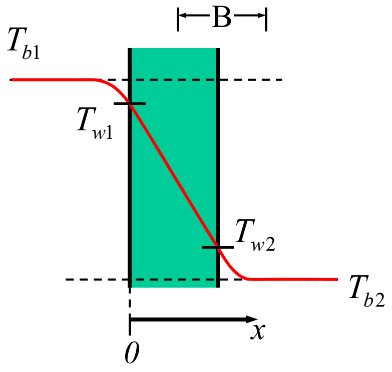
See handwritten notes in class.

https://pages.mtu.edu/~fmorriso/cm310/selected_lecture_slides.html

Example 4: Heat flux in a slab

Using the solution (conceptual):

For heat conduction in a slab with Newton's law of cooling boundary conditions, we sketched the solution as shown. **If only the heat transfer coefficient on the right side became infinitely large**, (no change in bulk temperatures) how would the sketch change? What are the predictions for ***T(x)*** and the **flux** for this case?



Let's try.

See handwritten notes in class.

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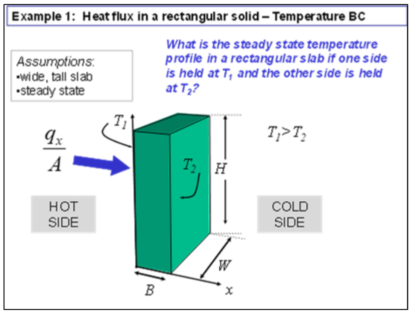
Heat transfer to:

- ✓ Slab
-
-



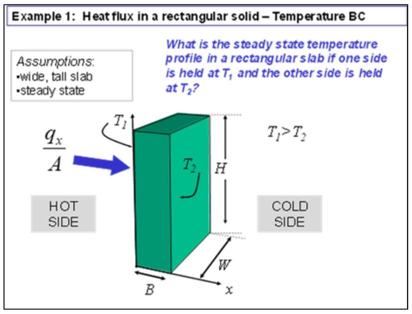
Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
•wide, tall slab
•steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?

Heat transfer to:

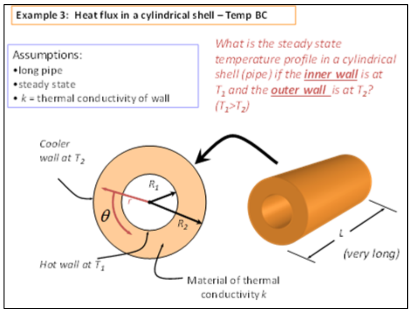
- ✓ Slab
- Cylindrical Shell
-



Example 1: Heat flux in a rectangular solid – Temperature BC

Assumptions:
•wide, tall slab
•steady state

What is the steady state temperature profile in a rectangular slab if one side is held at T_1 and the other side is held at T_2?



Example 3: Heat flux in a cylindrical shell – Temp BC

Assumptions:
•long pipe
•steady state
• k = thermal conductivity of wall

*What is the steady state temperature profile in a cylindrical shell (pipe) if the **inner wall** is at T_1 and the **outer wall** is at T_2? ($T_1 > T_2$)*