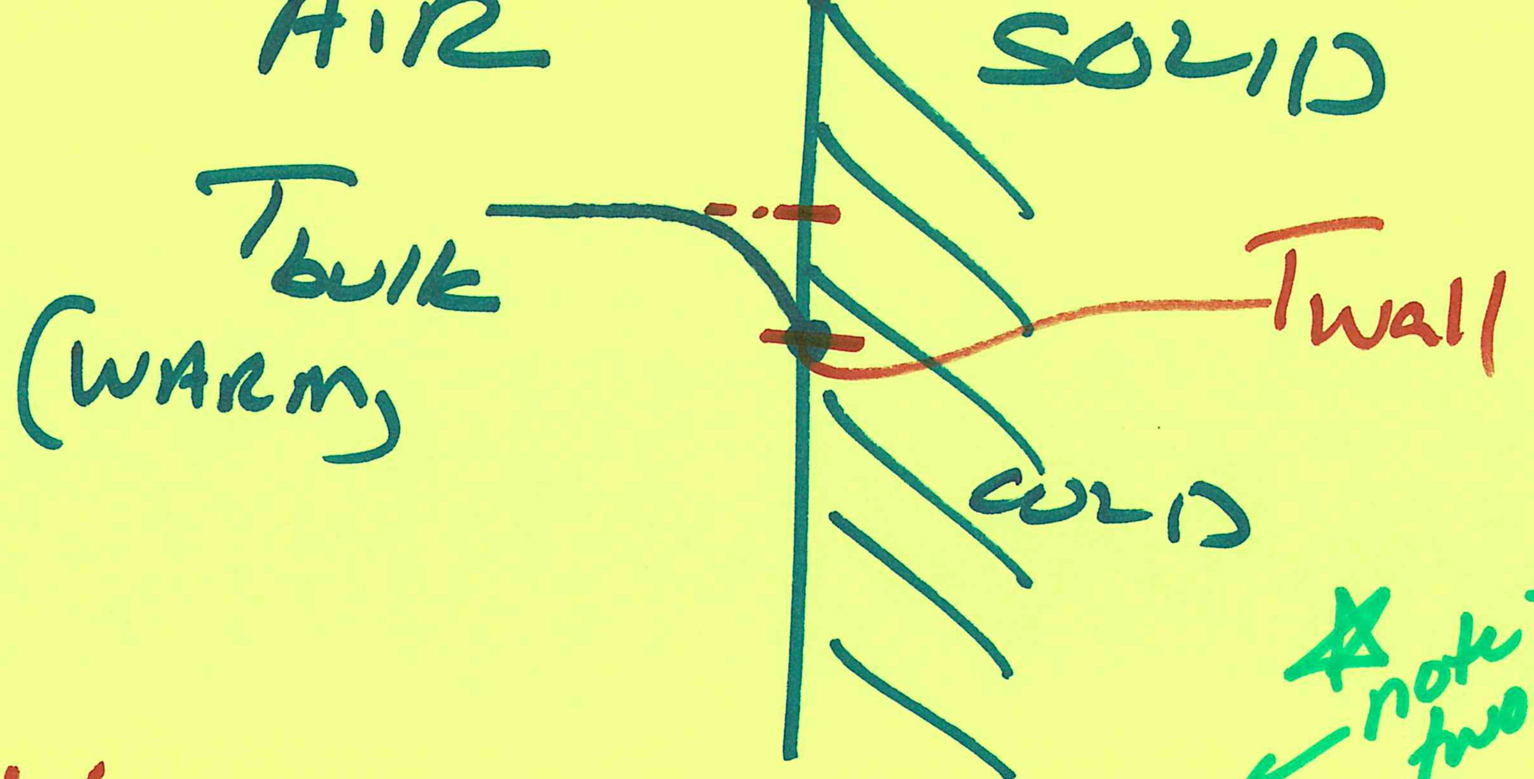
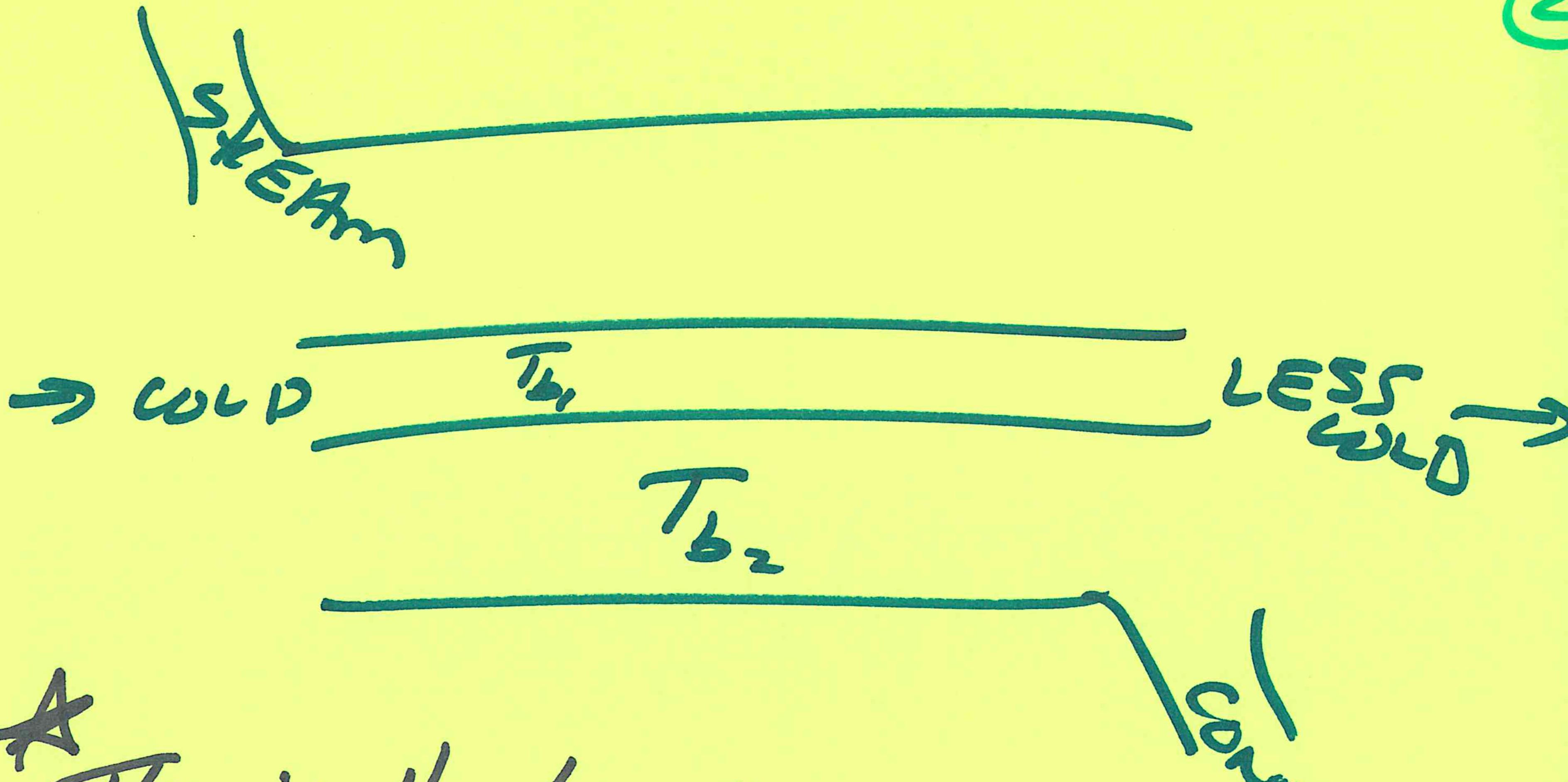
New Boundary Conditions:



$$\underbrace{\left|(T_{bulk} - T_{wall})h\right| = \left|\frac{q_x}{A}\right|}_{\text{Newton's LAW of COOLING}}$$

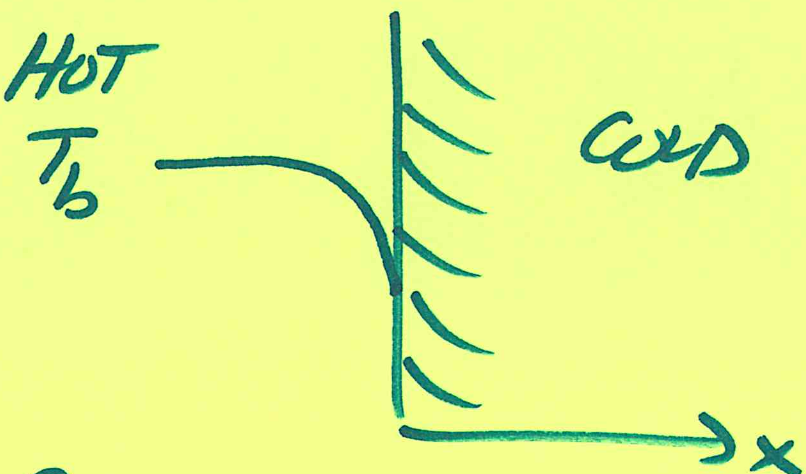
☆ note the two sets of absolute value signs

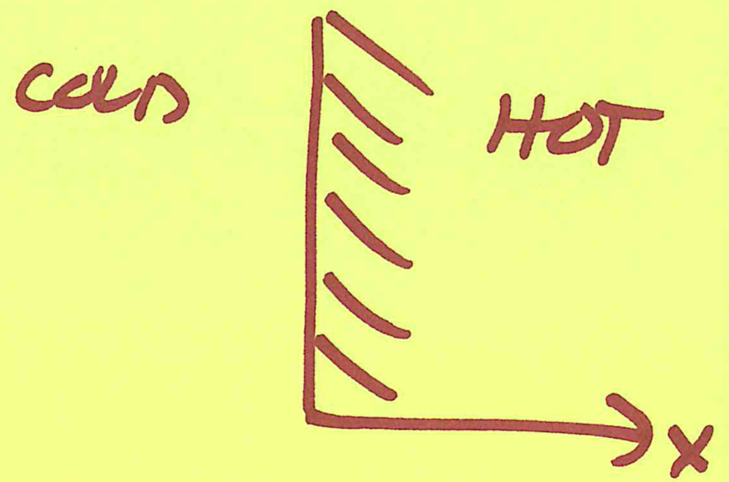


★ This is the boundary condition in a double-pipe heat exchanger

Newton's Law of Cooling

$\frac{q_x}{A}$ ← the coordinate system determines the sign of $\frac{q_x}{A}$



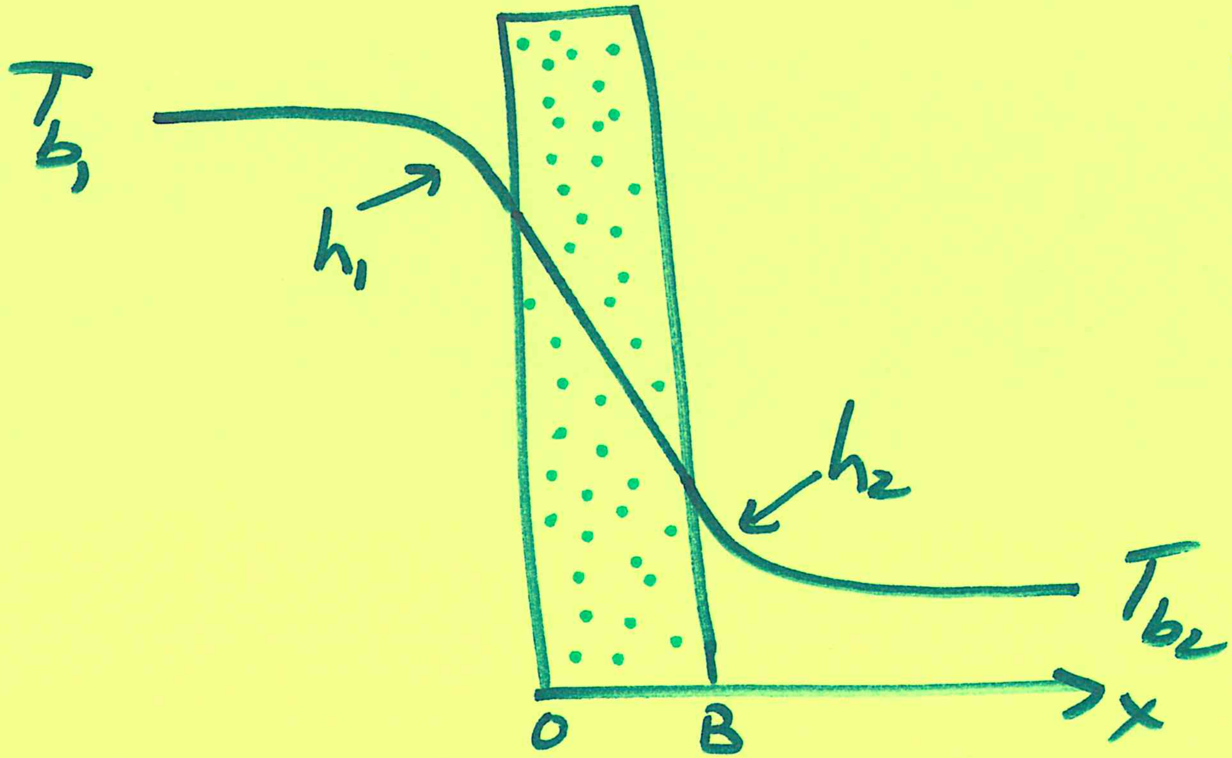


$\frac{q_x}{A} > 0$ ⟶ direction of flux is direction of increasing x

← dir of flux $\frac{q_x}{A} < 0$

Example 2: Steady, 1D rectangular heat conduction with Newton's Law of cooling BC

- steady
- long
- wide
- k



<u>Micro E bal</u> $\rightarrow$

SAME SOLN AS Ex 1, but diffrent BC:

$$T = c_1 x + c_2$$

★

BC: $x = 0 \quad T = T_{w_1}$

$x = B \quad T = T_{w_2}$

we do not know

What we do know:

$x = 0 \quad |T_{b_1} - T_{w_1}| h_1 = \left.\frac{q_x}{A}\right|_{\text{at } x=0} > 0$

$x = B \quad |T_{b_2} - T_{w_2}| h_2 = \left.\frac{q_x}{A}\right|_{\text{at } x=B} > 0$

For our system

Applying the BCs:

$$T = C_1 x + C_2$$

Fourier's LAW: $\frac{q_x}{A} = -k\frac{dT}{dx} = -kC_1$ at all x

ie. at $x=0$ & $x=B$

$x=0$ $\quad T = T_{w_1} = C_1(0) + C_2 = C_2$

$x=B$ $\quad T = T_{w_2} = C_1 B + C_2$

Newton's Law of Cooling

at $x=0$ $\quad \frac{q_x}{A} = -kC_1 = (T_{b_1} - T_{w_1})h_1$

$$\boxed{-kC_1 = (T_{b_1} - C_2)h_1}$$

one eqn, two unknowns

at $x = B$ $\quad \frac{q_x}{A} = -kc_1 = (T_{w_2} - T_{b_2})h_2$

substitute

$$\boxed{-kc_1 = h_2(c_1 B + c_2 - T_{b_2})}$$

second eqn in same two unknowns (solve).

Solving for c_1, c_2:

From 1st eqn, solve for c_2:

$$\boxed{c_2 = T_{b_1} + \frac{+kc_1}{h_1}}$$

now, substitute into second eqn

$$-kc_1 = h_2 c_1 B + h_2\left(T_{b_1} + \frac{kc_1}{h_1}\right) - h_2 T_{b_2}$$

solve for c_1:

$$c_1\left(h_2 B + \frac{kh_2}{h_1} + k\right) = -h_2 T_{b_1} + h_2 T_{b_2}$$

$$\boxed{c_1 = \frac{(T_{b_2} - T_{b_1})}{B + \frac{k}{h_1} + \frac{k}{h_2}}}$$

Substitute: $T = c_1 x + c_2 = T_{b_1} + c_1\left(\frac{k}{h_1} + x\right)$

$$T = \frac{(T_{b_2} - T_{b_1})}{\left(B + \frac{k}{h_1} + \frac{k}{h_2}\right)}\left(x + \frac{k}{h_1}\right) + T_{b_1}$$

$$\boxed{(T - T_{b_1}) = (T_{b_2} - T_{b_1})\left(\frac{\frac{x}{k} + \frac{1}{h_1}}{\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}}\right)}$$

Flux:

$$\boxed{\frac{q_x}{A} = -kG_1 = \frac{(T_{b_1} - T_{b_2})}{\left(\frac{1}{h_1} + \frac{B}{k} + \frac{1}{h_2}\right)}}$$

(matches slides)

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