Name: SOLUTION

Midterm Exam

CM 3110

October 25, 2007

SOLN

Please be neat.

Please write on only one side of each piece of paper in your solution.

Significant figures count.

1. (10 points) Does Newton's law of viscosity, shown below written in the $r\theta z$ coordinate system, hold for non-Newtonian fluids such as mayonnaise or molten plastics?

$$\tau_{rz} = -\mu\left(\frac{\partial v_z}{\partial r}\right)$$

ANSWER:

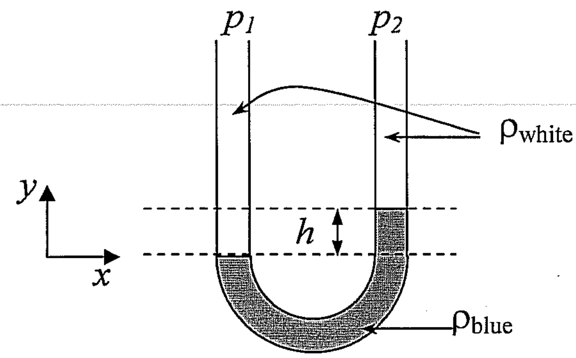
No. It only works for Newtonian fluids

2. (10 points) What is the definition of drag coefficient? Please give your answer in one or two sentences.

ANSWER:

$$C_D = \frac{F_{z,kinetic}}{\frac{\pi D^2}{4}\,\frac{1}{2}\rho v_\infty^2}$$ = a dimensionless kinetic force used in external flows

3. (5 points) In the manometer shown, what is the shear stress τ_{yx}?



ANSWER:

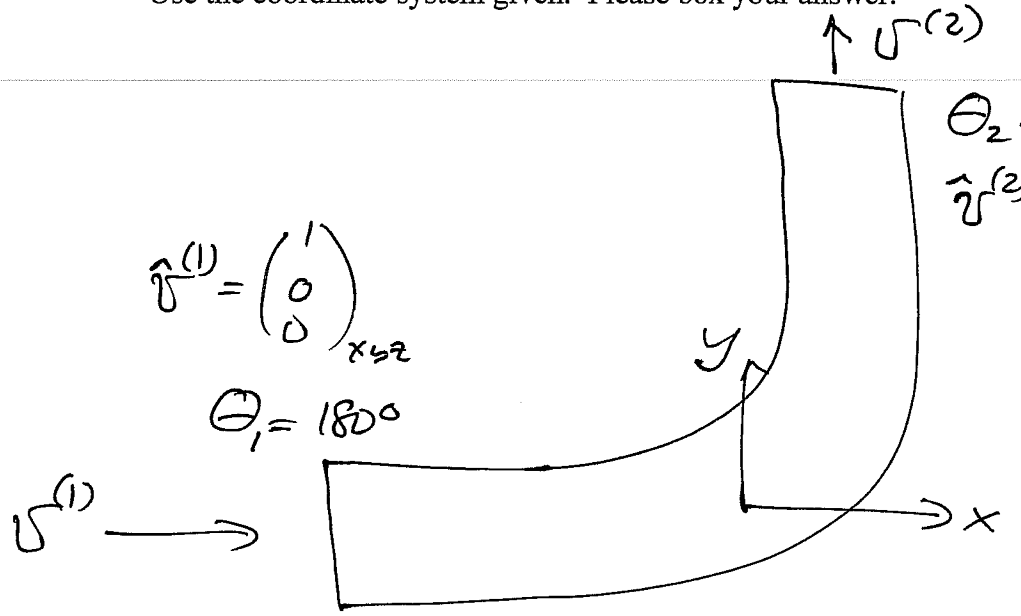
$\tau_{yx} = 0$ (no motion)

4. (15 points) In the microscopic momentum balance solution for steady flow in a round tube, what three assumptions lead to the conclusion that $\frac{\partial v_z}{\partial z} = 0$?

ANSWER:

① steady state
② flow only in z-direction
($v_x = v_y = 0$)
③ ρ = constant

5. (30 points) What is the vector force on the right-angle pipe bend shown below if the flow rate of water (density = 1000. kg/m^3; viscosity = 1.00 $mPa\ s$) is 1.84 gpm = 1.1609 x 10^{-4} m^3/s? The diameter of the pipe is 3.0 inches = 0.0762 m. You may assume that pressure does not change across the bend, and you may neglect gravity. Use the coordinate system given. Please box your answer.



$\theta_2 = 0$

$\hat{v}^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{xyz}$

$\hat{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$

$\theta_1 = 180^\circ$

$$0 = \frac{-\rho_1 A_1 \langle v^{(1)}\rangle^2 \cos\theta_1}{\beta_1}\hat{v}^{(1)} + \frac{-\rho_2 A_2 \langle v^{(2)}\rangle^2 \cos\theta_2}{\beta_2}\hat{v}^{(2)}$$

$$+ \cancel{\text{gravity}} + \cancel{\text{pressure}} + \underbrace{\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}_{xyz}}_{\substack{\text{force on} \\ \text{fluid} = \\ -\text{force on} \\ \text{bend}}}$$

told to omit

$\left.\begin{array}{l} \beta \sim 1 \text{ turbulent} \\ \beta = 0.75 \text{ laminar} \end{array}\right\}$ check

$$Re = \frac{\rho V D}{\mu} \qquad \langle V \rangle = \frac{Q}{\pi R^2}$$

$$\langle V^{(1)} \rangle = \langle V^{(2)} \rangle = \left(\frac{1.1609 \times 10^{-4}\, m^3}{s} \right) \frac{1}{\pi \left(\frac{.0762 m}{2} \right)^2}$$

$$\boxed{\langle V \rangle = 0.025456 \ \frac{m}{s}}$$

$$Re = \frac{\left(1000 \frac{kg}{\cancel{m^3}}\right)\left(0.025456 \frac{\cancel{m}}{\cancel{s}}\right)\left(.0762 \cancel{m}\right)}{.001 \cancel{Pa}\, s} \ \frac{\cancel{N}}{\cancel{Pa}\, m^2} \ \frac{\cancel{kg}\, \cancel{m}}{\cancel{N}\, \cancel{s^2}}$$

$$\boxed{Re = 1940} \Rightarrow \text{LAMINAR}$$

$$\beta_1 = 0.75$$
$$\beta_2 = 0.75$$

Note: $\rho_1 = \rho_2 = \rho$
$A_1 = A_2 = A = \pi R^2$

$$-\underline{F} = \underline{F}_{\substack{on\\bend}} = -\frac{\rho}{\beta} A \langle V \rangle^2 \Big(\underbrace{\cos\theta_1}_{-1} \hat{v}^{(1)} + \underbrace{\cos\theta_2}_{1} \hat{v}^{(2)} \Big)$$

$$\hat{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \qquad \hat{v}^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_{xyz}$$

$$= -\frac{\rho A \langle V \rangle^2}{\beta} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}_{xyz}$$

$$= \frac{\left(\frac{1000\,\cancel{kg}}{\cancel{m^3}}\right) \pi \left(\frac{0.0762\,\cancel{m}}{2}\right)^2 \left(0.025456\,\frac{\cancel{m}}{\cancel{s}}\right)^2}{0.75} \; \frac{N s^2}{\cancel{kg\,m}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}_{xyz}$$

$$\boxed{\underline{F}_{\substack{on\\bend}} = 3.9402 \times 10^{-3}\,N \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}_{xyz}}$$

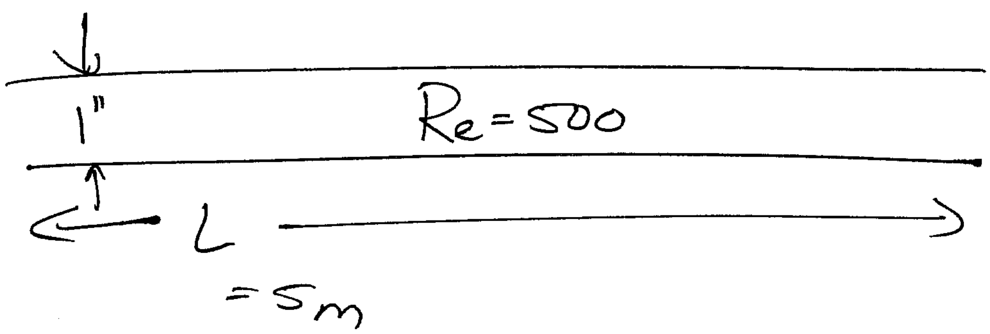
6. (30 points) Glycerin (density = 1258 kg/m^3, viscosity = 0.951 $Pa\ s$) flows in a horizontal pipe of length 5.0 m and diameter 1.0 in = 0.0254 m at a Reynolds number of 5.0 X 10^2.

a) What is the average velocity (m/s) of the glycerin in the tube?

b) What is the Fanning friction factor in the tube?

c) What is the pressure-drop (Pa) in the tube over the 5.0 m length?

d) What is the maximum velocity (m/s) of the glycerin in the tube?



From the notes, the velocity profile is

$$V_z(r) = \frac{R^2 \Delta P}{4 \mu L}\left[1 - \left(\frac{r}{R}\right)^2\right]$$ Lecture 5)

$$V_{max} = V_z(0) = \frac{R^2 \Delta P}{4 \mu L}$$

$$V_{av} = \frac{V_{max}}{2} = \frac{R^2 \Delta P}{8 \mu L}$$

V_{av} is also in Re

$$500 = Re = \frac{\rho V_{av} D}{\mu} = \frac{\left(1258 \frac{\cancel{kg}}{m^3}\right) V_{av} (0.0254 \cancel{m})}{\frac{\cancel{kg}\,\cancel{m}}{\cancel{N} s^{\cancel{2}}}\, 0.951\ \cancel{Pa}\, \cancel{s}\, \frac{\cancel{N}}{\cancel{Pa}\, m^{\cancel{2}}}}$$

$$\boxed{V_{av} = 14.88\ \frac{m}{s} \Big| = 15\ m/s \quad 2\ \text{sig figs}}$$

$$\boxed{V_{max} = 30\ m/s}$$

$$f = \frac{16}{Re} \text{ (laminar)} = \boxed{0.032 = f}$$

calc ΔP from V_{av}: $V_{av} = R^2 \Delta P / 8 \mu L$

$$14.88\ \frac{m}{s} = \frac{\left(\frac{0.0254\ m}{2}\right)^2 \Delta P}{8\ (.951\ Pa\,s)(5\ m)}$$

$$\boxed{\Delta P = 3.5 \times 10^6\ Pa = 3.5\ MPa}\ //$$