MIDTERM
SOLN
CM3110
2009

1. τ_{yz} is the flux of z-momentum in the y-direction

$$\boxed{\tau_{yz} = -\mu \frac{dV_z}{dy}}$$

2. $$D_H = \frac{4 \text{ cross-sectional area}}{\text{wetted perimeter}}$$

$$\boxed{D_H = \frac{4(\pi R_2^2 - \pi R_1^2)}{2\pi R_1 + 2\pi R_2}}$$

3. $Re = 2100$ is largest Reynolds # for laminar flow

$$2100 = \frac{\rho \langle v \rangle D}{\mu}$$

$$\langle v \rangle = \frac{2100\,\mu}{\rho D}$$

$$= \frac{(2100)\left(6.005 \times 10^{-4} \frac{\cancel{lbm}}{\cancel{ft}\, s}\right)}{\left(62.25 \frac{\cancel{lbm}}{ft^{\cancel{3}}}\right)\left(\frac{0.315\, \cancel{in}}{12\, \cancel{in}/\cancel{ft}}\right)}$$

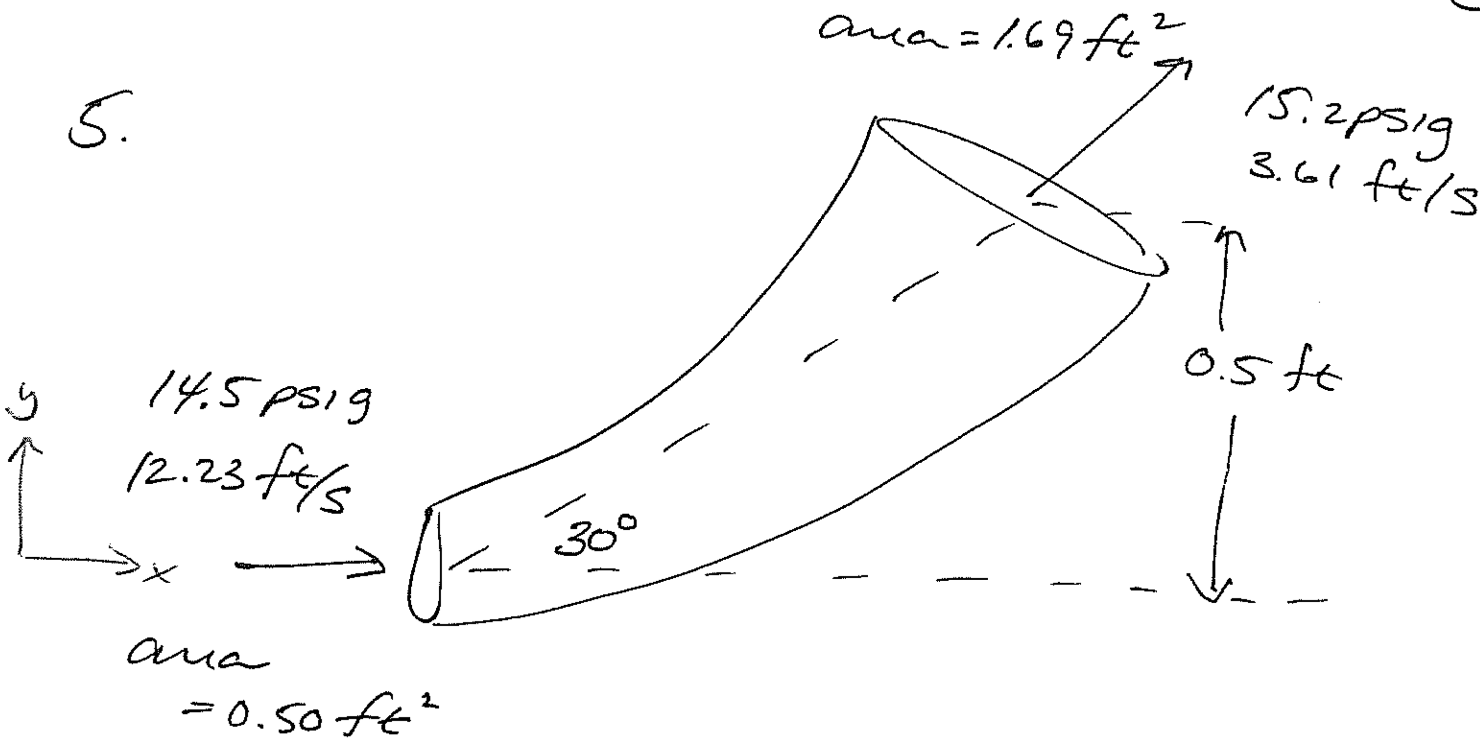
$$\boxed{\langle v \rangle = 0.77\ ft/s}$$

4. $$\langle V \rangle = \frac{\int_0^{2\pi}\int_0^{R} V_z(r)\, r\,dr\,d\theta}{\pi R^2}$$

$$= \frac{1}{\cancel{\pi} R^2}\, 2\cancel{\pi} \int_0^{R} A\left(1-\frac{r}{R}\right)^{\frac{1}{7}} r\,dr$$

$$\boxed{\langle V \rangle = \frac{2A}{R^2}\int_0^{R}\left(1-\frac{r}{R}\right)^{\frac{1}{7}} r\,dr}$$

5.



MACRO MOMENTUM BAL

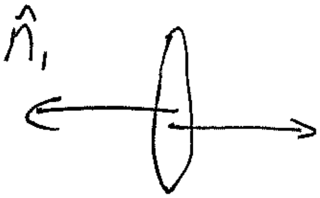
$$0 = \frac{-\rho A_1 \langle V^{(1)} \rangle^2 \cos\theta}{\beta_1} \hat{r}^{(1)} + \frac{-\rho A_2 \langle V^{(2)} \rangle^2 \cos\theta_2}{\beta_2} \hat{r}^{(2)} + \cancel{\underline{F}_{gravity}} + \underline{F}_{walls} + \underline{F}_{pressure}$$

$\beta_1 = \beta_2 = 1$ turbulent

$\theta_1 = \measuredangle$ between $\hat{n}_1$ and $\hat{v}^{(1)} = 180^\circ$

$$\boxed{\cos\theta_1 = -1}$$

$$\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = \hat{n}_1$$



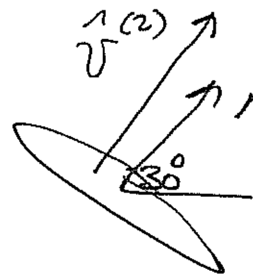
$$\hat{v}^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

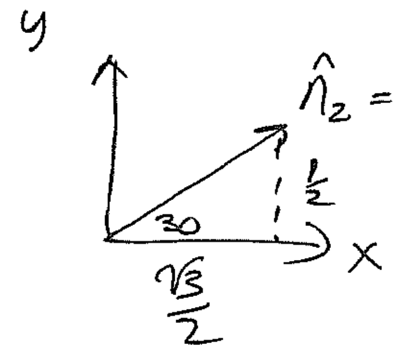
$\theta_2 = \measuredangle$ between $\hat{n}_2$ and $\hat{v}^{(2)} = 0$

$$\boxed{\cos\theta_2 = 1}$$

$\hat{v}^{(2)}$

$\hat{n}_2 =$





$$\hat{n}_2 = \hat{v}^{(2)} = \begin{pmatrix} \cos 30^\circ \\ \sin 30^\circ \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

$$\rho = 997.08 \text{ kg/m}^3 = 62.25 \frac{\text{lbm}}{\text{ft}^3}$$

$$\underline{F}_{pressure} = P_1 A_1 \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\hat{n}_1} + P_2 A_2 \underbrace{\begin{pmatrix} -\sqrt{3}/2 \\ -0.5 \\ 0 \end{pmatrix}}_{-\hat{n}_2}$$

$$-\underline{F}_{walls} = -\rho A_1 V_1^2 (-1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \rho A_2 V_2^2 \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix} \qquad + \begin{pmatrix} P_1 A_1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} P_2 A_2 \sqrt{3}/2 \\ P_2 A_2 (0.5) \\ 0 \end{pmatrix}$$

$$F_y = \rho A_2 V_2^2 (0.5) + \frac{P_2 A_2}{2}$$

$$F_y = \left(\frac{\cancel{8}\,7 \text{ lb}_f}{32.174 \frac{\cancel{ft}}{\cancel{lbm}}}\right)\left(\frac{62.25 \cancel{lbm}}{\cancel{ft^3}}\right)\left(\underline{1.69 \cancel{ft^2}}\right)\left(\frac{3.61 \cancel{ft}}{8}\right)^2\left(\frac{1}{2}\right)$$

$$+ \left(\frac{15.2 \text{ lbf}}{\cancel{in^2}}\right)\left(\frac{12 \cancel{in}}{\cancel{ft^2}}\right)^2\left(1.69 \cancel{ft^2}\right)\left(\frac{1}{2}\right)$$

$$F_y = \frac{1}{32.174}\left(685.505\right) + 1849.5 \quad (lb_f)$$

$$F_y = (21.3 + 1849.5)\, lb_f$$

$$F_y = 1870.8\, lb_f$$

$$\boxed{F_y = 1900\, lb_f \quad (2 \text{ sig figs})}$$

The force on the control volume (the fluid) from the presence of the walls is 1900 lb_f upward.