Name: SOLUTION

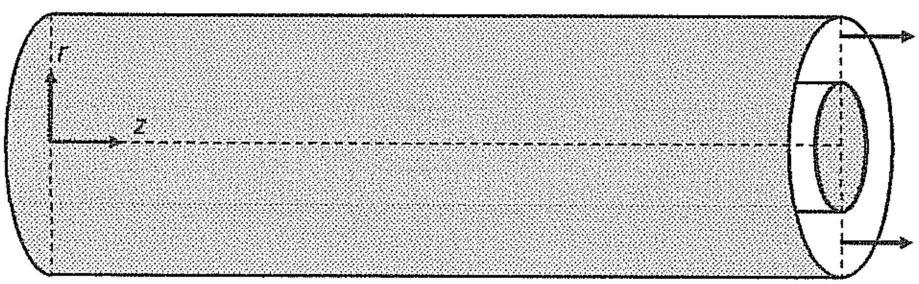
Box #: ____________

Mini-Exam II

CM 3110 8 October 2009

Note:
Significant figures count.
Please box your final answers.
Please be neat.

1. (50 points) For the apparatus shown in the figure below, an incompressible, Newtonian fluid is made to flow under an axial pressure gradient in the gap between two concentric cylinders. The flow is steady, and the inlet pressure is P_0 and the outlet pressure is P_L. The tube is of length L and is horizontal. You may neglect gravity. The microscopic mass balance (continuity equation) and the microscopic momentum balance (Navier-Stokes equations) are shown on the next page. For <u>each</u> term in <u>all four</u> equations, cross out the terms that are zero and give a reason for each decision.



Side view: steady flow in annulus between cylinders

See next page

$$\underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{\text{steady}} + \frac{1}{r}\underbrace{\cancel{\frac{\partial(\rho r v_r)}{\partial r}}}_{v_r=0} + \frac{1}{r}\underbrace{\cancel{\frac{\partial(\rho v_\theta)}{\partial \theta}}}_{v_\theta=0} + \frac{\partial(\rho v_z)}{\partial z} = 0 \quad \Rightarrow \quad \boxed{\frac{\partial v_z}{\partial z}=0}$$

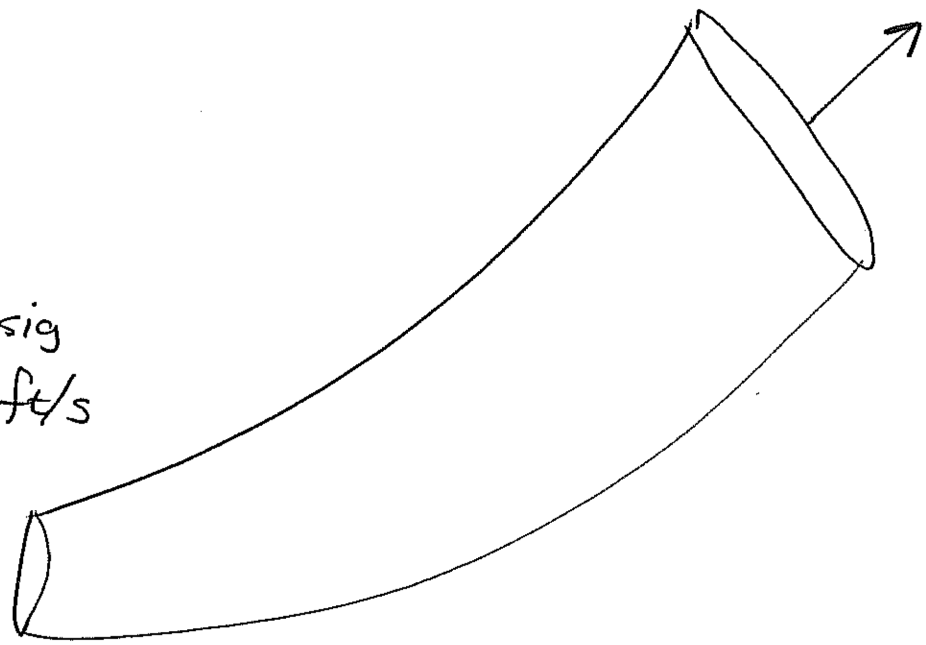
$$\rho\left(\underbrace{\cancel{\frac{\partial v_r}{\partial t}}}_{\text{steady state}} + \underbrace{\cancel{v_r\frac{\partial v_r}{\partial r}}}_{V_r=0} + \underbrace{\cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}}}_{V_\theta=0 \text{ or } V_r=0} - \underbrace{\cancel{\frac{v_\theta^2}{r}}}_{V_\theta=0} + \underbrace{\cancel{v_z\frac{\partial v_r}{\partial z}}}_{V_r=0}\right) = -\frac{\partial P}{\partial r} + \mu\left(\underbrace{\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_r)}{\partial r}\right)}}_{V_r=0} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}}}_{V_r=0} - \underbrace{\cancel{\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}}}_{V_\theta=0} + \underbrace{\cancel{\frac{\partial^2 v_r}{\partial z^2}}}_{V_r=0}\right) + \underbrace{\cancel{\rho g_r}}_{\text{neglect}}$$

$$\rho\left(\underbrace{\cancel{\frac{\partial v_\theta}{\partial t}}}_{\text{steady state}} + \underbrace{\cancel{v_r\frac{\partial v_\theta}{\partial r}}}_{V_\theta=0 \text{ or } V_r=0} + \underbrace{\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}}_{V_\theta=0} + \underbrace{\cancel{\frac{v_r v_\theta}{r}}}_{V_r=0 \text{ or } V_\theta=0} + \underbrace{\cancel{v_z\frac{\partial v_\theta}{\partial z}}}_{V_\theta=0}\right) = -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left(\underbrace{\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(r v_\theta)}{\partial r}\right)}}_{V_\theta=0} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}}_{V_\theta=0} + \underbrace{\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}}_{V_r=0} + \underbrace{\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}}_{V_\theta=0}\right) + \underbrace{\cancel{\rho g_\theta}}_{\text{neglect}}$$

$$\rho\left(\underbrace{\cancel{\frac{\partial v_z}{\partial t}}}_{\text{steady state}} + \underbrace{\cancel{v_r\frac{\partial v_z}{\partial r}}}_{V_r=0} + \underbrace{\cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}}}_{V_\theta=0} + \underbrace{\cancel{v_z\frac{\partial v_z}{\partial z}}}_{\text{continuity}}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}}}_{\theta\text{-symmetry}} + \underbrace{\cancel{\frac{\partial^2 v_z}{\partial z^2}}}_{\text{continuity}}\right) + \underbrace{\cancel{\rho g_z}}_{\text{no z component to } \underline{g}}$$

2.

$P_1 = 14.5$ psig
$\langle V \rangle_1 = 12.23$ ft/s
$z_1 = 0$



$P_2 = ?$
$\langle V \rangle_2 = 3.61 \frac{ft}{s}$
$z_2 = 0.5$ ft

Mechanical energy balance:

$$\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2\alpha} + g\Delta z + \cancel{F} = \cancel{\frac{-W_{s,by}}{m}}$$

$\boxed{\alpha = 1}$

told to neglect

no work

$$\frac{P_2 - P_1}{\rho} + \frac{\langle V \rangle_2^2 - \langle V \rangle_1^2}{2} + g(z_2 - z_1) = 0$$

$$\boxed{P_2 = \rho\left[(z_1 - z_2)g + \frac{\langle V \rangle_1^2 - \langle V \rangle_2^2}{2}\right] + P_1}$$

$$P_2 = \left(\frac{62.25\ lb_m}{ft^3}\right)\left[-(0.5\ ft)\left(\frac{32.174\ ft}{s^2}\right) + \left[\underbrace{\left(12.23\ \frac{ft}{s}\right)^2}_{149.5729} - \underbrace{\left(3.61\ \frac{ft}{s}\right)^2}_{13.0321}\right]\frac{1}{2}\right] + 14.5\ psi$$

$$P_2 = \left(\frac{62.25\ lb_m}{ft^3}\right)\left[-16.087\ \frac{ft^2}{s^2} + 68.2704\ \frac{ft^2}{s^2}\right] + 14.5\ \frac{lb_f}{in^2}$$

$$= \underbrace{3248.8524\ \frac{\cancel{lb_m}}{\cancel{ft}\ \cancel{s^2}}\ \frac{\cancel{s^2}\ lb_f}{32.174\ \cancel{ft}\ \cancel{lb_m}}\ \frac{\cancel{ft^2}}{(12\ in)^2}}_{0.70113} + 14.5\ \frac{lb_f}{in^2}$$

$$\boxed{P_2 = 15.2\ psi}$$