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Mini Exam 2
Solution

1. $Q = 0.053 \frac{ft^3}{s}$ $\langle V \rangle = ?$

$$\langle V \rangle = \frac{Q}{\pi R^2}$$

$$= \frac{0.053\,ft^3}{s} \frac{(12\,in/ft)^2}{\pi (2\,in)^2}$$

$$= 6.073 \quad \frac{ft}{s}$$

$$= \boxed{6.1\ ft/s}$$

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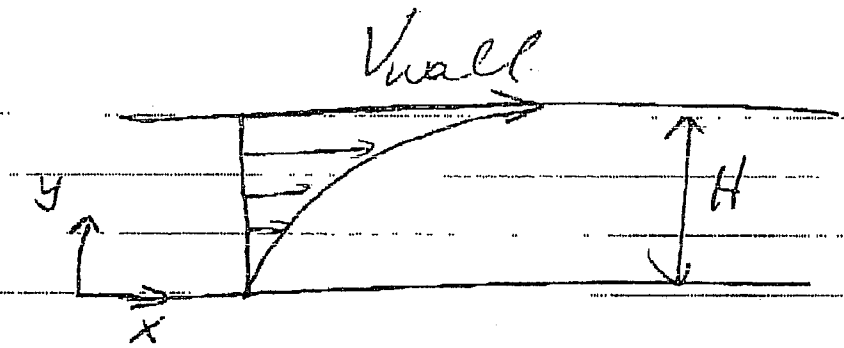
2



(a) flux of x-momentum in y-dir is negative

(b) continuity equation

$$\underline{v} = \begin{pmatrix} v_x \\ 0 \\ 0 \end{pmatrix}_{xyz} \qquad \begin{array}{l} \text{steady} \\ \rho = \text{constant} \end{array}$$

$$\Rightarrow \boxed{\frac{\partial v_x}{\partial x} = 0}$$

equation of motion
with above assumptions +
gravity neglected:

X-component

$$0 = -\frac{\partial P}{\partial x} + \mu \left(\frac{\partial^2 v_x}{\partial y^2} + \cancel{\frac{\partial^2 v_x}{\partial z^2}} \right)$$

no side walls

y-component:

$$\boxed{0 = -\frac{\partial P}{\partial y}}$$

z-component

$$\boxed{0 = -\frac{\partial P}{\partial z}}$$

Solving the x-component

$$0 = \frac{\Delta P}{L} + \mu \frac{d^2 V_x}{dy^2}$$

$$\frac{-\Delta P}{\mu L} = \frac{d^2 V_x}{dy^2}$$

$$\boxed{\frac{dV_x}{dy} = \frac{-\Delta P}{\mu L} y + C_1}$$

need C_1

$$V_x = \frac{-\Delta P}{2\mu L} y^2 + C_1 y + C_2$$

Boundary conditions

$y=0 \quad v_x = 0 \Rightarrow C_2 = 0$
$y=H \quad v_x = V_{wall}$

$$V_{wall} = \frac{-\Delta P}{2\mu L} H^2 + C_1 H$$

$$C_1 = \left[V_{wall} + \frac{\Delta P H^2}{2\mu L} \right] \frac{1}{H}$$

stress: Newton's Law of Visc

$$\tau_{yx} = -\mu \frac{dv_x}{dy}$$

$$= -\mu \left[\frac{-\Delta P}{\mu L} y + C_1 \right]$$

n.i.c. $$\tau_{yx} = -\mu \left[\frac{-\Delta P}{\mu L} y + \frac{V_{wall}}{H} + \frac{\Delta P H}{2\mu L} \right]$$

$$\boxed{\tau_{yx} = \frac{\Delta P}{L} y - \frac{V_{wall}\mu}{H} - \frac{\Delta P H}{2L}}$$