Types of Heat Transfer

- Conduction (Fourier’s Law)
  \[ \frac{q_x}{A} = -k \frac{dT}{dx} \]
- Forced convection (due to flow)
  \[ \mathbf{v} \cdot \nabla T \]
- Source terms
- Free convection (fluid motion due to density variations brought on by temperature differences)
  \[ \frac{Dv^*}{Dt} = \left( \nabla^2 v_j \right)^* + \text{Gr}^* \]
- Heat transfer with phase change (e.g. condensing fluids)
  \[ \Delta H_{\text{vap}} \]

last subject in the course

{radiation}

Heat transfer due to radiation

- In atoms and molecules electrons can exist in multiple, discrete energy states
- Transfers between energy states are accompanied by an emission of radiation

Energy

\[ \text{discrete energy levels} \]

Quantum Mechanics

© Faith A. Morrison, Michigan Tech U.
Radiation versus Conduction and Convection

Continuum view
- Conduction is caused by macroscopic temperature gradients
- Convection is caused by macroscopic flow
- Radiation? NO CONTINUUM EXPLANATION

Molecular view
- Conduction?
- Convection?
- Radiation is caused by changes in electron energy states in molecules and atoms

There is, of course, a molecular explanation of these effects, since we know that matter is made of atoms and molecules.

Continuum versus Molecular description of matter

A continuum is infinitely divisible

Real matter is not a continuum; at small enough length scales, molecules are discrete.

© Faith A. Morrison, Michigan Tech U.
Individual molecules carry:
• chemical identity
• macroscopic velocity (speed and direction)
• internal energy (Brownian velocity)

When they undergo Brownian motion within an inhomogeneous mixture, they cause:
• diffusion (mass transport)
• exchange of momentum (momentum transport)
• conduction (energy transport)

© Faith A. Morrison, Michigan Tech U.

**Kinetic Theory**  J. C. Maxwell, L. Boltzmann, 1860

- Molecules are in constant motion (Brownian motion)
- Temperature is related to $E_{k,av}$ of the molecules

**Simplest model**
- no particle volume
- no intermolecular forces

**More realistic model**
- finite particle volume
- intermolecular forces

© Faith A. Morrison, Michigan Tech U.
Kinetic Theory

Is based on Brownian motion (molecules in constant motion proportional to their temperature)

Predicts that properties that are carried by individual molecules (chemical identity, momentum, average kinetic energy) will be transported **down** gradients in these quantities.

\[ \Rightarrow \text{Transport laws are due to Brownian motion} \]

Heat Transfer by Radiation

Is due to the release of energy stored in molecules that is **not** related to average kinetic energy (temperature), but rather to the population of excited states.

\[ \Rightarrow \text{Radiation is NOT a Brownian effect} \]

Radiation

* does not require a medium to transfer energy (works in a vacuum)
* travels at the speed of light, \( c = 3 \times 10^{10} \text{ cm/s} \)
* travels as a wave; differs from x-rays, light, only by wavelength, \( \lambda \)
* radiation is important when temperatures are high

\[ \frac{q}{A} \propto T^4 \]

**examples:**
- the sun
- home radiator
- hot walls in vacuum oven
- heat exchanger walls when \( \Delta T \) is high and a vapor film has formed

© Faith A. Morrison, Michigan Tech U.
Why does radiation flux scale with temperature, which is related to average kinetic energy?

As a molecule gains energy, it both speeds up (increases average kinetic energy) and increases its population of excited states.

The increase in average kinetic energy is reflected in temperature (directly proportional).

The increase in number of electrons in excited states is reflected in increased radiation flux. Electrons enter excited states in proportion to $T^4$.
What causes energy transfer by radiation?

- Energy hits surface
- Pushes some molecules into an excited state
- When the molecules/atoms relax from the excited state, they emit radiation

\[ q_{\text{emit}} \propto T^4 \]

\[ \alpha = \text{absorptivity} \]
\[ \alpha \equiv \frac{q_{\text{absorbed}}}{q_{\text{incident}}} < 1 \]

Absorption

In general, \( \alpha \) is a function of wavelength

\[ \alpha = \alpha(\lambda) \]

Gray body: a body for which \( \alpha \) is constant (does not depend on \( \lambda \))
Black body: a body for which \( \alpha = 1 \), i.e. absorbs all incident radiation
Emission

**Gray Body**: a body for which $\alpha$ is constant

**Black Body**: a body for which $\alpha = 1$

Kirchhoff’s Law: emissivity equals absorptivity at the same temperature

$$\alpha = \varepsilon$$

the fraction of energy absorbed by a material = the relative amount of energy emitted from that material compared to a black body

Black Bodies

**Stefan-Boltzmann Law**: the amount of energy emitted by a black body is proportional to $T^4$

$$\frac{q_{\text{emitted, black body}}}{A} = \sigma T^4$$

$$\sigma = 0.1712 \times 10^{-8} \frac{BTU}{h^2 R^2}$$

$$= 5.676 \times 10^{-8} \frac{W}{m^2 K^4}$$

© Faith A. Morrison, Michigan Tech U.
Non-Black Bodies

\[ \varepsilon = \text{emissivity} \]
\[ \varepsilon = \frac{q_{\text{emitted}}}{q_{\text{emitted, black body}}} \]
\[ \frac{q_{\text{emitted, non-black body}}}{A} = \varepsilon \frac{q_{\text{emitted, black body}}}{A} = \varepsilon \sigma T^4 \]

Energy emitted by a non-black body

\[ \frac{q_{\text{emitted, non-black body}}}{A} = \varepsilon \sigma T^4 \]

Stefan-Boltzmann:

\[ \frac{q_{\text{emitted, black body}}}{A} = \sigma T^4 \]

How does this relate to chemical engineering?

Consider a furnace with an internal blower:

There is heat transfer due to convection:

\[ q_{\text{convection}} = h_{\text{conv}} A(T_s - T_b) \]

There is also heat transfer due to radiation:

\[ q_{\text{radiation}} = h_{\text{rad}} A(T_s - T_b) \]

\[ q_{\text{total}} = q_{\text{conv}} + q_{\text{rad}} \]

© Faith A. Morrison, Michigan Tech U.
Where do we get $h_{\text{rad}}$?

object in furnace:

$q_{\text{emitted, non-black body}} = A\varepsilon|_{T_s} \sigma T_b^4$

$q_{\text{absorbed}} = \alpha|_{T_s} A\sigma T_s^4 = A\varepsilon|_{T_s} \sigma T_s^4$

energy emitted by walls, which are acting as a black body

net energy absorbed:

$q_{\text{transferred to body}} = A\varepsilon|_{T_s} \sigma \left(T_s^4 - T_b^4\right)$

assuming $\varepsilon|_{T_s} \approx \varepsilon|_{T_b}$

Finally, calculate $h_{\text{rad}}$

net energy absorbed:

$q_{\text{transferred to body}} = A\varepsilon|_{T_s} \sigma \left(T_s^4 - T_b^4\right)$

assuming $\varepsilon|_{T_s} \approx \varepsilon|_{T_b}$

equating with expression for $h$:

$A\varepsilon|_{T_s} \sigma \left(T_s^4 - T_b^4\right) = h_{\text{rad}} A(T_s - T_b)$

$h_{\text{rad}} = \frac{\sigma \varepsilon|_{T_s} \left(T_s^4 - T_b^4\right)}{T_s - T_b}$

Geankoplis 4th ed., eqn 4.10-10 p304
Example: Geankoplis 4.10-3

A horizontal oxidized steel pipe carrying steam and having an OD of 0.1683 m has a surface temperature of 374.9 K and is exposed to air at 297.1 K in a large enclosure. Calculate the heat loss for 0.305 m of pipe from natural convection plus radiation. For the steel pipe, use an emissivity of 0.79.

Radiation Heat Transfer Between Two Infinite Plates

Consider a quantity of radiation energy that is emitted from surface 1.

See: Geankoplis, section 4.11B
Also: Bird, Stewart, and Lightfoot, “Transport Phenomena” 1960 Wiley PP446-448
### First round – Surface 2

**Quantity of energy incident at surface 2:**

\[
\frac{q_{1-2}}{A} = \varepsilon_1 \sigma T_1^4
\]

**Quantity of energy absorbed at surface 2:**

\[
\alpha_2 \left( \frac{q_{1-2}}{A} \right) A = \varepsilon_2 \left( \varepsilon_1 \sigma T_1^4 \right) A
\]

\[
\alpha_2 = \varepsilon_2
\]

**Quantity of energy reflected from surface 2:**

\[
(1 - \varepsilon_2) \left( \varepsilon_1 A \sigma T_1^4 \right)
\]

This energy goes back to surface 1.

---

### Second round – Surface 1

**Quantity of energy absorbed at surface 1 (second round):**

\[
\varepsilon_1 \left[ \left(1 - \varepsilon_2\right) \left( \varepsilon_1 A \sigma T_1^4 \right) \right]
\]

**Quantity of energy reflected from surface 1 (second round):**

\[
(1 - \varepsilon_1) \left[ \left(1 - \varepsilon_2\right) \left( \varepsilon_1 A \sigma T_1^4 \right) \right]
\]

© Faith A. Morrison, Michigan Tech U.
Radiation Heat Transfer Between Two Infinite Plates

Quantity of energy absorbed at surface 2 (third round):

\[ \varepsilon_2 \left[ (1 - \varepsilon_1)(1 - \varepsilon_2) \left( \varepsilon_1 A \sigma T_1^4 \right) \right] \]

Quantity of energy reflected from surface 2 (third round):

\[ (1 - \varepsilon_2) \left[ (1 - \varepsilon_1)(1 - \varepsilon_2) \left( \varepsilon_1 A \sigma T_1^4 \right) \right] \]

Third round – surface 2

There is a pattern.

Now, calculate the radiation energy going from surface 1 to surface 2:

\[ q_{1\rightarrow2} = \sum \left( \text{energy absorbed at surface 2} \right) \]

\[ = \varepsilon_2 \left( \varepsilon_1 A \sigma T_1^4 \right) \]

\[ + \varepsilon_2 (1 - \varepsilon_1)(1 - \varepsilon_2) \left( \varepsilon_1 A \sigma T_1^4 \right) \]

\[ + \varepsilon_2 (1 - \varepsilon_1)^2 (1 - \varepsilon_2)^2 \left( \varepsilon_1 A \sigma T_1^4 \right) \]

\[ \ldots + \varepsilon_2 (1 - \varepsilon_1)^n (1 - \varepsilon_2)^n \left( \varepsilon_1 A \sigma T_1^4 \right) + \ldots \]

Later, calculate energy from 2 to 1; then subtract to obtain net energy transferred.

© Faith A. Morrison, Michigan Tech U.
Radiation Heat Transfer Between Two Infinite Plates

Radiation energy going from surface 1 to surface 2:

\[ q_{1-2} = \varepsilon_1 \varepsilon_2 A \sigma T_1^4 \sum_{n=0}^{\infty} (1 - \varepsilon_1)^n (1 - \varepsilon_2)^n \]

How can we calculate \( \sum_{n=0}^{\infty} x^n \) ?

Answer: \( S = \frac{1}{1-x} \)

---

Radiation energy going from surface 1 to surface 2:

\[ q_{1-2} = \frac{\varepsilon_1 \varepsilon_2 A \sigma T_1^4}{1 - [(1 - \varepsilon_1)(1 - \varepsilon_2)]} \]

\[ = \frac{\varepsilon_1 \varepsilon_2 A \sigma T_1^4}{1 - [1 - \varepsilon_1 - \varepsilon_2 + \varepsilon_1 \varepsilon_2]} = \frac{\varepsilon_1 \varepsilon_2 A \sigma T_1^4}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2} \]

\[ \frac{q_{1-2}}{A} = \frac{\sigma T_1^4}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \]

Final Result

© Faith A. Morrison, Michigan Tech U.
Radiation Heat Transfer Between Two Infinite Plates

Radiation energy going from surface $1$ to surface $2$:

$$\frac{q_{1\rightarrow 2}}{A} = \frac{\sigma T_1^4}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Radiation energy going from surface $2$ to surface $1$:

$$\frac{q_{2\rightarrow 1}}{A} = \frac{\sigma T_2^4}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

NET Radiation energy going from surface $1$ to surface $2$:

$$\frac{q_{1\rightarrow 2} - q_{2\rightarrow 1}}{A} = \frac{\sigma \left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

Radiation Shields

Purpose of Heat Shields:
To reduce the amount of energy transfer from (hotter) plate at $T_1$ to second (cooler) plate at $T_3$.

Note:
$$q_{net,1\rightarrow 2} = q_{net,2\rightarrow 3} = q$$
Analysis of Radiation Shields

We will assume that the emissivity is the same for all surfaces.

\[
q_{net,1 \rightarrow 2} = \frac{\sigma \left(T_1^4 - T_2^4\right)}{A} \left(\frac{1}{\varepsilon} + \frac{1}{1 - \varepsilon}\right)
\]

\[
q_{net,2 \rightarrow 3} = \frac{\sigma \left(T_2^4 - T_3^4\right)}{A} \left(\frac{1}{\varepsilon} + \frac{1}{1 - \varepsilon}\right)
\]

Now we eliminate \(T_2\) between these equations.

Note:
\[
q_{net,1 \rightarrow 2} = q_{net,2 \rightarrow 3} = q
\]

© Faith A. Morrison, Michigan Tech U.
Analysis of Radiation Shields

**1 Heat Shield**

\[
\frac{q}{A} = \left( \frac{1}{2} \right) \frac{\sigma (T_1^4 - T_3^4)}{\left( \frac{2}{\varepsilon} - 1 \right)}
\]

With one heat shield present, \( q \) falls by half compared to no heat shield.

---

**N Heat Shields**

\[
\frac{q}{A} = \left( \frac{1}{N + 1} \right) \frac{\sigma (T_1^4 - T_3^4)}{\left( \frac{2}{\varepsilon} - 1 \right)}
\]

With \( N \) heat shields present, \( q \) falls by a factor of \( 1/N \) compared to no heat shield.

© Faith A. Morrison, Michigan Tech U.