(reducing bend)

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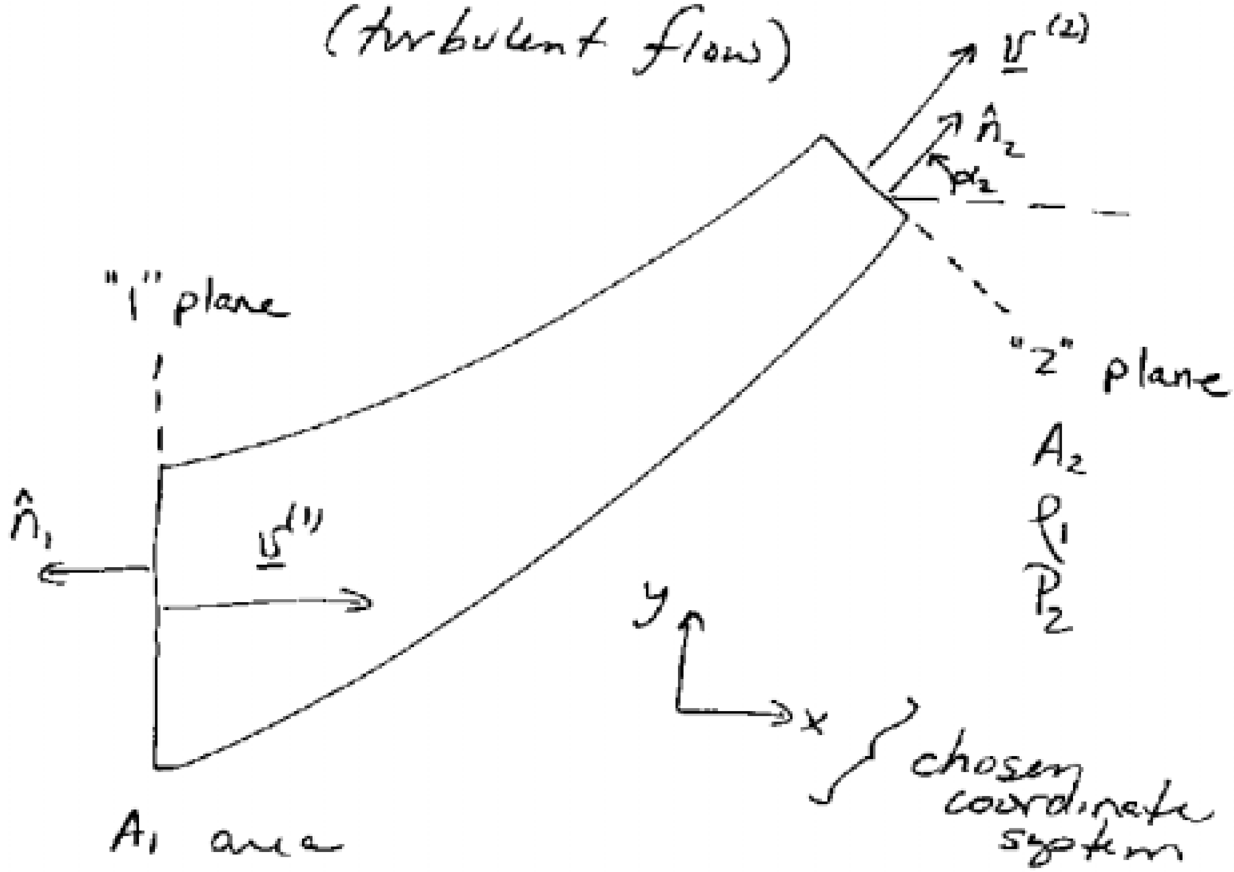
9-29-200

FAM

CALCULATE THE FORCE ON A REDUCING BEND
(turbulent flow)



A_1 area

ρ_1 density

P_1 pressure

We seek $\underline{F}_{WALL} = -\underline{F}_R$

$\underline{F}_R$ = vector force on the fluid by walls

this force appears in the <u>macroscopic</u> momentum <u>balance</u>

(We could also get it from integrating microscopic forces, but that's much harder.)

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MACROSCOPIC MASS BALANCE

$$M_1 = M_2$$

$$\langle v^{(1)} \rangle \rho_1 A_1 = \langle v^{(2)} \rangle \rho_2 A_2$$

MACROSCOPIC MOMENTUM BALANCE

$$-\left\{ \hat{u}_1 \left(\frac{m_1 \cos\theta_1 \langle v^{(1)} \rangle}{\beta_1} \right) + \hat{u}_2 \left(\frac{M_2 \cos\theta_2 \langle v^{(2)} \rangle}{\beta_2} \right) \right\}$$

$$+ \sum \underline{F}_{\substack{\text{on}\\ \text{fluid}}} = \underbrace{0}_{\text{steady state}}$$

We need to identify all the symbols in the convective terms and all the forces.

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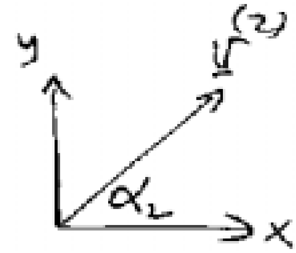

$$\hat{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} = \hat{\imath}$$

$$\hat{v}_2 = \begin{pmatrix} \cos\alpha_2 \\ \sin\alpha_2 \\ 0 \end{pmatrix}_{xyz} = \hat{\imath}\cos\alpha_2 + \hat{\jmath}\sin\alpha_2$$

$$\cos\theta_1 = \hat{v}_1 \cdot \hat{n}_1 = -1$$

$$\cos\theta_2 = \hat{v}_2 \cdot \hat{n}_2 = 1$$

$\beta_1 = \beta_2 = 1$ (turbulent)

<u>FORCES:</u> pressure, gravity, wall forces

FORCE <u>ON</u> DUE TO PRESSURE ON SURFACE "1"

$$= \begin{pmatrix} P_1 A_1 \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

pressure force is in direction of velocity

$$= P_1 A_1 \hat{v}_1$$

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FORCE ON DUE TO PRESSURE ON SURFACE "2" $= -P_2 \ A_2 \ \hat{U}_2$

pressure is in direction of velocity

pressure is fluid in the bend pushing on fluid outside of bend. To make it force on fluid in bend, need negative

$$= \begin{pmatrix} -P_2 A_2 \cos\alpha_2 \\ -P_2 A_2 \sin\alpha_2 \\ 0 \end{pmatrix}_{xyz}$$

FORCE ON DUE TO GRAVITY $= \begin{pmatrix} 0 \\ -M_{total}\, g \\ 0 \end{pmatrix}_{xyz} = -M_{total}\, g \ \hat{j}$

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FORCE ON FLUID DUE TO WALLS $= \begin{pmatrix} F_{Rx} \\ F_{Ry} \\ F_{Rz} \end{pmatrix}_{xyz} = \underline{F}_R$

Now, assemble the MACRO MOMENTUM BALANCE:

$$0 = -\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{xyz} \langle v^{(1)} \rangle^2 \rho_1 A_1 (-1) + \begin{pmatrix} \cos\alpha_2 \\ \sin\alpha_2 \\ 0 \end{pmatrix}_{xyz} \langle v^{(2)} \rangle^2 \rho_2 A_2 (+1) \right\}$$

$$+ \begin{pmatrix} P_1 A_1 \\ 0 \\ 0 \end{pmatrix}_{xyz} + \begin{pmatrix} -P_2 A_2 \cos\alpha_2 \\ -P_2 A_2 \sin\alpha_2 \\ 0 \end{pmatrix}_{xyz} + \begin{pmatrix} 0 \\ -M_{total}\, g \\ 0 \end{pmatrix}_{xyz} + \begin{pmatrix} F_{Rx} \\ F_{Ry} \\ F_{Rz} \end{pmatrix}_{xyz}$$

z-component: $\boxed{F_{Rz} = 0}$

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x-component:

$$-\langle v^{(1)}\rangle^2 \rho_1 A_1 + \cos\alpha_2 \langle v^{(2)}\rangle^2 \rho_2 A_2 - P_1 A_1 + P_2 A_2 \cos\alpha_2 = F_{Rx}$$

y-component:

$$\sin\alpha_2 \langle v^{(2)}\rangle^2 \rho_2 A_2 + P_2 A_2 \sin\alpha_2 + M_{tot}\, g = F_{Ry}$$

WE NOW HAVE $\underline{F_R} = \begin{pmatrix} F_{Rx} \\ F_{Ry} \\ F_{Rz} \end{pmatrix}$; SINCE $\underline{F}_{WALL} = -\underline{F}_R$,

$$F_{WALL} = \begin{pmatrix} \langle v^{(1)}\rangle^2 \rho_1 A_1 - \cos\alpha_2 \langle v^{(2)}\rangle^2 \rho_2 A_2 + P_1 A_1 - P_2 A_2 \cos\alpha_2 \\ -\sin\alpha_2 \langle v^{(2)}\rangle^2 \rho_2 A_2 - P_2 A_2 \sin\alpha_2 - \underbrace{M_{tot}\, g}_{\text{usually neglected.}} \\ 0 \end{pmatrix}_{xyz}$$