

The Power-Law, Generalized Newtonian Constitutive Equation

Ref: Faith A. Morrison, *Understanding Rheology* (Oxford University Press: New York, 2001)

Cartesian coordinates

$$\begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix}_{xyz} = \eta \underline{\underline{\dot{\gamma}}} \quad 1-1$$

$$\eta \equiv m \left(\frac{1}{2} \cdot \begin{array}{l} \text{sum of squares} \\ \text{of each term in } \underline{\underline{\dot{\gamma}}} \end{array} \right)^{\frac{n-1}{2}} = m \left(\frac{1}{2} \cdot \sum_{p=1}^3 \sum_{j=1}^3 \dot{\gamma}_{pj}^2 \right)^{\frac{n-1}{2}} \quad 1-2$$

$$\underline{\underline{\dot{\gamma}}} = \begin{pmatrix} 2 \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} & \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \\ \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} & 2 \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} & \frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} & 2 \frac{\partial v_z}{\partial z} \end{pmatrix}_{xyz} \quad 1-3$$

Cylindrical coordinates

$$\begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{pmatrix}_{r\theta z} = \eta \underline{\underline{\dot{\gamma}}} \quad 2-1$$

$$\eta \equiv m \left(\frac{1}{2} \cdot \begin{array}{l} \text{sum of squares} \\ \text{of each term in } \underline{\underline{\dot{\gamma}}} \end{array} \right)^{\frac{n-1}{2}} = m \left(\frac{1}{2} \cdot \sum_{p=1}^3 \sum_{j=1}^3 \dot{\gamma}_{pj}^2 \right)^{\frac{n-1}{2}} \quad 2-2$$

$$\underline{\underline{\dot{\gamma}}} = \begin{pmatrix} 2 \frac{\partial v_r}{\partial r} & r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} & \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \\ r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} & 2 \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) & \frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \\ \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} & \frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} & 2 \frac{\partial v_z}{\partial z} \end{pmatrix}_{r\theta z} \quad 2-3$$

Spherical coordinates

$$\begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{r\phi} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta\phi} \\ \tau_{\phi r} & \tau_{\phi\theta} & \tau_{\phi\phi} \end{pmatrix}_{r\theta\phi} = \eta \underline{\underline{\dot{\gamma}}} \quad 3-1$$

$$\eta \equiv m \left(\frac{1}{2} \cdot \begin{array}{l} \text{sum of squares} \\ \text{of each term in } \underline{\underline{\dot{\gamma}}} \end{array} \right)^{\frac{n-1}{2}} = m \left(\frac{1}{2} \cdot \sum_{p=1}^3 \sum_{j=1}^3 \dot{\gamma}_{pj}^2 \right)^{\frac{n-1}{2}} \quad 3-2$$

$$\underline{\underline{\dot{\gamma}}} = \begin{pmatrix} 2 \frac{\partial v_r}{\partial r} & r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} & \frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) \\ r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} & 2 \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) & \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left(\frac{v_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} \\ \frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) & \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left(\frac{v_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} & 2 \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right) \end{pmatrix}_{r\theta\phi} \quad 3-3$$