Homework 2  
CM4650  
Spring 2015 corrected 9Feb2015

Part A: Problems 1-4 are Due Wednesday 28 January 2015, in class  
Part B: Problems 5-7 are Due Wednesday 4 February 2015, in class

Please do not write on the back side of any page of your solution. Please write legibly and large. You may find this page helpful in this homework: www.chem.mtu.edu/~fmorriso/cm310/MorrisonCoverMatter(c)2011.pdf. Please make note of the discussion about the stress sign convention and the difference between the two symbols $\Pi$ (Understanding Rheology) and $\Pi$ (An Introduction to Fluid Mechanics).

Note that the book has some typos: www.chem.mtu.edu/~fmorriso/cm4650/URerrata.html

**Part A**

1. (20 points) For the vectors given below, what are the following quantities equal to? Show your work in Einstein notation before substituting the specific vectors $\vec{a}$ and $\vec{b}$ from below.  
   a. the gradient of $\vec{a}$, $\nabla \vec{a}$  
   b. the divergence of $\vec{b}$, $\nabla \cdot \vec{b}$  
   c. the Laplacian of $\vec{b}$, $\nabla \cdot \nabla \vec{b} = \nabla^2 \vec{b}$  
   d. $\vec{a} - \vec{b}$

   $\begin{align*}
   \vec{a} &= \left( \begin{array}{c}
   3x^2y - 6z \\
   2xyz^2 \\
   2y + 6z 
   \end{array} \right)_{xyz} \\
   \vec{b} &= (2xy^2 - 4yz)\hat{e}_x + 3x^2y\hat{e}_y - 7\hat{e}_z
   \end{align*}$

2. (10 points) What is $A_{lp}v_p\hat{e}_l$ in Gibbs (vector-tensor) notation? What is $\frac{\partial r_{jm}}{\partial x_j}\hat{e}_m$ in Gibbs notation?
3. (10 points) The flow rate through a finite surface $S$ can be written as:

$$Q = \int \left[ \hat{n} \cdot \mathbf{v} \right]_{surf} dS$$

where $\hat{n}$ is the unit normal to the surface $dS$ and $\mathbf{v}$ is the velocity at $dS$. The solution for the velocity field in Poiseuille flow in a slit is given in the text (see example 3.52). Starting with the equation above, calculate the flow rate through the slit cross-section. Show your work.

4. (10 points) The fluid force $\mathbf{F}$ on a finite surface $S$ can be written as:

$$\mathbf{F} = \int \left[ \hat{n} \cdot \left( \mathbf{\Pi} \right) \right]_{surf} dS$$

where $\hat{n}$ is the unit normal to the surface $dS$ and $\mathbf{\Pi}$ is the total stress tensor with the sign convention of our text. The solution for the velocity field in Poiseuille flow in a slit is given in the text (see example 3.52). Starting with the equation above, calculate the total vector force (three components) on upper wall. Show your work.

**Part B**

5. (10 points) Sketch (by hand is all I require; you can use a computer if you want) the following vector velocity field ($x, y, z$ in millimeters):

$$\mathbf{v} \left( \frac{mm}{s} \right) = \left( \begin{array}{c} -3x \\ 6y \\ -3z \end{array} \right)_{xyz}$$

You may confine yourself to the first quadrant ($x, y, z$ all positive) and the plane where $y = 0$.

Hint: You will need to choose some points, calculate $v = |\mathbf{v}|$ and the direction of $\mathbf{v}$ at those points, and then draw arrows of the appropriate lengths and directions at the points. Usually we center the vector at the points chosen.

6. (20 points) Text 3.15 (Tilted Poiseuille slit). Do not use tables for the momentum balance; use Einstein notation as in the text, begin from Gibbs notation (do not use tables).

7. (20 points) Text 3.19 (Axial drag flow of a wire; you may use tables)