Homework 3
CM4650
Spring 2015

Due: Wednesday 18 February 2015, in class

Please do not write on the back side of the pages. Please write legibly and large. Thank you.

1. (10 points) Using Einstein notation, show that the following identity holds:

\[(A \cdot B)^T = B^T \cdot A^T\]

2. (10 points) What is the rate of deformation \( \dot{\gamma} \equiv |\dot{\gamma}| \) for the flow given below? Recall that \( \dot{\gamma} \equiv \nabla \mathbf{v} + (\nabla \mathbf{v})^T \). Please show your calculation. To what standard flow category does this velocity field belong? (did not mention this standard flow in lecture; see text)

\[
\mathbf{v}(mm/s) = \begin{pmatrix}
-0.173s^{-1}x_1 \\
0 \\
0.173s^{-1}x_3
\end{pmatrix}_{123}
\]

3. (10 points) For the flow shown below, calculate the stress generated by a Newtonian fluid of viscosity \( \mu = 2.93 \times 10^2 \text{ cp} \) subjected to that flow. What is the shear rate \( \dot{\gamma} \)? What is the shear stress generated (in \( \text{Pa} \))? What are the values of \( N_1 \) and \( N_2 \) generated?

\[
\mathbf{v}(mm/s) = \begin{pmatrix}
(3.2s^{-1})x_2 \\
0 \\
0
\end{pmatrix}_{123}
\]

4. (20 points) For the tensors given below, what are the values of the three invariants? Use the invariant definitions in Chapter 2.

\[
A = \begin{pmatrix}
8 & 2 & 0 \\
2 & 0 & 0 \\
0 & 0 & -8
\end{pmatrix}_{123}
\]

\[
B = \begin{pmatrix}
4 & 0.8x_2 & 0 \\
0.8x_2 & 4x_1 & 0 \\
0 & 0 & -8
\end{pmatrix}_{123}
\]

5. (20 points) In problem 3.18 in the text, the torsional flow between concentric cylinders is described. Beginning with the general expression for torque given
below, write the integrals that will be needed for calculating the torque necessary to turn the inner cylinder. What is the direction of the torque vector? Calculate the torque vector for this problem. Vector cross product is discussed beginning on page 26. Hint: once you know what \( R \) and \( \hat{n} \) are, write \( \Pi \) in general terms to see which coefficients you will need to calculate the torque. That way you will not spend time evaluating terms that are not needed for the torque. The solution for velocity in this problem is given below; pressure is a function of \( r \) only, \( p = p(r) \).

\[
T = \iint_A [R \times (\hat{n} \cdot (-\Pi))]_{\text{surface}} dA
\]

\[
v_\theta = \frac{\kappa^2}{\kappa^2 - 1} (R \Omega) \left( \frac{r}{R} - \frac{R}{r} \right)
\]