1. (20 points) Use Einstein notation to write this expression in terms of Cartesian components: $\nabla \cdot (u \, v)$. Note that both $u$ and $v$ are variables (depend on position). Expand any derivatives of products in your final answer. Insert the appropriate summation signs in your final answer.

2. (20 points) Write the following Einstein notation expression in Gibbs notation (vector/tensor/del notation): $\frac{\partial w_k}{\partial x_k}$

3. (20 points) A fluid is placed in between two long, wide plates and made to flow at steady state. A constant pressure gradient (pressure higher upstream than downstream) is imposed. In addition, the top wall moves at a speed $V$ parallel to the bottom wall (see figure). We model the flow and arrive at the following differential equation for the balance of $x_1$-momentum:

$$\frac{\partial p}{\partial x_1} = \mu \frac{\partial^2 v_1}{\partial x_2^2}$$

a. Solve the differential equation for $v_1(x_2)$; include any needed integration constants.

b. What are the velocity boundary conditions you will use to evaluate the integration constants? Express the boundary conditions mathematically. You do not need to solve for the integration constants.
4. (20 points) (20 points) A steady upward flow of an incompressible, Newtonian fluid is created between two very long, wide parallel plates as shown in the figure. The upstream pressure is \( P_0 \) and the pressure a distance \( L \) downstream is \( P_L \). You may **not** neglect gravity, which acts against the pressure driving force. Calculate the velocity field as a function of position in this flow. Please show your work and indicate your assumptions. Express your final answer only in terms of quantities given in the problem or figure.
5. (20 points) The torsional parallel plate rheometer measures rheological properties by subjecting a disk of fluid to a rotational flow (see figure). The velocity field is a function of both \( z \) and \( r \). We are interested in calculating the torque required to turn the upper plate at an angular rate of \( \Omega \) (\( rad/s \)). The velocity field is given by:

\[
v = \begin{pmatrix} 0 \\ v_\theta(r, z) \\ 0 \end{pmatrix}_{r\theta z}
\]

\[
T_{on} = \int_S [\mathbf{R} \times (\mathbf{n} \cdot -\mathbf{\Pi})]_{surface} dS
\]

For a Newtonian, incompressible fluid in torsional parallel plate flow, identify the quantities in the equation for torque \( T_{on} \) (see above). Be specific. Your answer will be the equation for torque with all the quantities written in terms of the geometry, or \( v_\theta(r, z) \), or other quantities given in the problem. Specify the limits of the integrals.