

2nd Exam Formulas

Polymer Rheology Prof. Faith Morrison

$$\text{Tensor magnitude: } A = |\underline{\underline{A}}| = +\sqrt{\frac{\underline{\underline{A}}:\underline{\underline{A}}}{2}}$$

$$\text{Rate of deformation: } \dot{\gamma} = |\dot{\underline{\underline{\gamma}}}|$$

Shear strain:

$$\gamma_{21}(t_a, t_b) = \int_{t_a}^{t_b} \dot{\gamma}_{21}(t') dt'$$

$$\text{Navier-Stokes Equation: } \rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p + \mu \nabla^2 \underline{v} + \rho \underline{g}$$

Cauchy Momentum Equation (equation of motion, EOM):

$$\rho \left(\frac{\partial \underline{v}}{\partial t} + \underline{v} \cdot \nabla \underline{v} \right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

$$\text{Continuity Equation: } \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \underline{v})$$

$$\text{Fluid force on a surface S: } \underline{f} = \iint_S [\hat{n} \cdot \underline{\underline{\Pi}}]_{\text{surface}} dA$$

$$\text{Flow rate through surface S: } Q = \iint_S [\hat{n} \cdot \underline{v}]_{\text{surface}} dA$$

$$\text{Fluid torque on a surface S: } \underline{T} = \iint_S \underline{R} \times [\hat{n} \cdot \underline{\underline{\Pi}}]_{\text{surface}} dA \quad (\underline{R} = \text{lever arm})$$

$$\text{Newtonian, incompressible fluid: } \underline{\underline{\tau}} = -\mu (\nabla \underline{v} + (\nabla \underline{v})^T)$$

$$\text{Generalized Newtonian fluid (GNF): } \underline{\underline{\tau}} = -\eta(\dot{\gamma}) \dot{\underline{\underline{\gamma}}}$$

$$\text{Power-law GNF model: } \eta(\dot{\gamma}) = m \dot{\gamma}^{n-1}$$

(Note that m and n are parameters of the model and are constants)

$$\text{Carreau-Yasuda GNF model: } \eta(\dot{\gamma}) = \eta_{\infty} + (\eta_o - \eta_{\infty}) \left[1 + (\dot{\gamma} \lambda)^a \right]^{\frac{n-1}{a}}$$

(Note that a , λ and n , η_o , and η_{∞} are parameters of the model and are constants)

Elongational flow (uniaxial, biaxial): $\underline{v} = \begin{pmatrix} -\frac{\dot{\epsilon}(t)}{2} x_1 \\ -\frac{\dot{\epsilon}(t)}{2} x_2 \\ \dot{\epsilon}(t) x_3 \end{pmatrix}_{123}$

Shear flow: $\underline{v} = \begin{pmatrix} \dot{\zeta}(t) x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$

Steady shearing kinematics: $\dot{\zeta}(t) = \dot{\gamma}_0$

Start-up of steady shearing kinematics: $\dot{\zeta}(t) = \begin{cases} 0 & t < 0 \\ \dot{\gamma}_0 & t \geq 0 \end{cases}$

Cessation of steady shearing kinematics: $\dot{\zeta}(t) = \begin{cases} \dot{\gamma}_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$

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Steady shear viscosity: $\eta = \frac{-(\tau_{21})}{\dot{\gamma}_0}$

Steady elongational viscosity: $\bar{\eta} = \frac{-(\tau_{33} - \tau_{11})}{\dot{\epsilon}_0}$
