EXAM 1

CM4650

14 FEB 08

1. Convection: $\underline{v} \cdot \nabla \underline{v}$

2. $\underline{a} \cdot \nabla \underline{b} = a_i \hat{e}_i \cdot \frac{\partial b_j}{\partial x_p} e_p \hat{e}_j$

δ_{ip}

"i becomes p"

$$= \boxed{a_p \frac{\partial b_j}{\partial x_p} \hat{e}_j}$$

3. $u_k A_{jk} \frac{\partial w_p}{\partial x_j} \hat{e}_p$

$$= \boxed{\sum_{k=1}^{3} \sum_{j=1}^{3} \sum_{p=1}^{3} u_k A_{jk} \frac{\partial w_p}{\partial x_j} \hat{e}_p}$$

4. $\underline{\underline{A}} = \nabla \underline{v} + (\nabla \underline{v})^T$

to add, put on common index: 1st = i second = j

$$\nabla \underline{v} = \frac{\partial v_m}{\partial x_p} \underset{\overset{\|}{i}}{\hat{e}_p} \underset{\overset{\|}{j}}{\hat{e}_m}$$

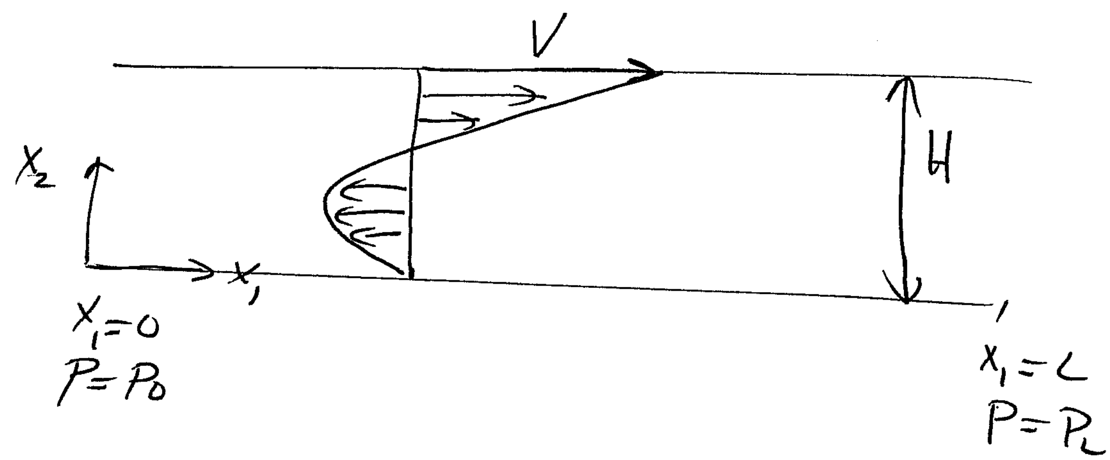
$$(\nabla \underline{v})^T = \frac{\partial v_m}{\partial x_p} \underset{\overset{\|}{i}}{\hat{e}_m} \underset{\overset{\|}{j}}{\hat{e}_p}$$

$$\nabla \underline{v} + (\nabla \underline{v})^T = \frac{\partial v_j}{\partial x_i} \hat{e}_i \hat{e}_j + \frac{\partial v_i}{\partial x_j} \hat{e}_i \hat{e}_j$$

$$= \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) \hat{e}_i \hat{e}_j$$

21-component $\Rightarrow$ $i = 2$, $j = 1$

$$\boxed{[\underline{\underline{A}}]_{21} = \left(\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} \right)}$$



$$\rho \left(\cancel{\frac{\partial \underline{v}}{\partial t}} + \cancel{(\underline{v} \cdot \nabla \underline{v})} \right) = -\nabla P + \mu \nabla^2 \underline{v} + \cancel{\rho \underline{g}}$$

steady state   unidirectional flow   neglect

$$\nabla \cdot \underline{v} = 0 \qquad \underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$0 = \frac{\partial}{\partial x_i} \overbrace{e_i \cdot v_p \hat{e}_p}^{\delta_{ip}}$$

"i becomes p"

$$= \frac{\partial v_p}{\partial x_p} = \sum_{p=1}^{3} \frac{\partial v_p}{\partial x_p} = \frac{\partial v_1}{\partial x_1} + \cancel{\frac{\partial v_2}{\partial x_2}} + \cancel{\frac{\partial v_3}{\partial x_3}}$$

$v_2 = 0$ $v_3 = 0$

$$\Rightarrow \boxed{\frac{\partial v_1}{\partial x_1} = 0}$$

$$\nabla^2 \underline{v} = \nabla \cdot \nabla \underline{v} = \frac{\partial}{\partial x_p} \hat{e}_p \cdot \frac{\partial}{\partial x_m} \hat{e}_m v_s \hat{e}_s$$

δ_{pm} "p becomes m"

$$= \frac{\partial}{\partial x_m} \frac{\partial}{\partial x_m} v_s \hat{e}_s$$

$0 = \nabla \cdot \underline{v}$

wide

$$\nabla^2 \underline{v} = \begin{pmatrix} \cancel{\frac{\partial^2 v_1}{\partial x_1^2}} + \frac{\partial^2 v_1}{\partial x_2^2} + \cancel{\frac{\partial^2 v_1}{\partial x_3^2}} \\ \cancel{\frac{\partial^2 v_2}{\partial x_1^2}} + \cancel{\frac{\partial^2 v_2}{\partial x_2^2}} + \cancel{\frac{\partial^2 v_2}{\partial x_3^2}} \\ \cancel{\frac{\partial^2 v_3}{\partial x_1^2}} + \cancel{\frac{\partial^2 v_3}{\partial x_2^2}} + \cancel{\frac{\partial^2 v_3}{\partial x_3^2}} \end{pmatrix}_{123} \begin{array}{l} \\ v_2 = 0 \\ v_3 = 0 \end{array}$$

$$\nabla^2 \underline{v} = \begin{pmatrix} \frac{\partial^2 v_1}{\partial x_2^2} \\ 0 \\ 0 \end{pmatrix}_{123}$$

EQUATION OF MOTION:

$$0 = -\nabla p + \mu \nabla^2 \underline{v}$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} -\frac{\partial P}{\partial X_1} \\ -\frac{\partial P}{\partial X_2} \\ -\frac{\partial P}{\partial X_3} \end{pmatrix}_{123} + \mu \begin{pmatrix} \frac{\partial^2 v_1}{\partial X_2^2} \\ 0 \\ 0 \end{pmatrix}_{123}$$

2-component: $\boxed{\frac{\partial P}{\partial X_2} = 0}$

3-component: $\boxed{\frac{\partial P}{\partial X_3} = 0}$

$\Rightarrow P = P(X_1)$ only

1-component: $\boxed{\frac{\partial P}{\partial X_1} = \mu \frac{\partial^2 v_1}{\partial X_2^2}}$ ⓐ

note: $v_1 = v_1(X_2)$ only due to continuity $(\nabla \cdot \underline{v} = 0)$ and wide plate assumption

$$\underbrace{\frac{dP}{dx_1}}_{f(x_1)} = \underbrace{\mu \frac{d^2 V_1}{dx_2^2}}_{g(x_2)} = \lambda$$

$\Rightarrow$ equal to same constant.

$$\frac{dP}{dx_1} = \lambda$$

$$\boxed{P = \lambda x_1 + C_1}$$

(2a) $P = P_0 \quad x_1 = 0$

$P = P_L \quad x_1 = L$

(3) $$\boxed{P = \left(\frac{P_L - P_0}{L}\right) x_1 + P_0}$$

$$\lambda = \left(\frac{P_L - P_0}{L}\right)$$

$$\mu \frac{d^2 V_1}{dx_2^2} = \lambda$$

$$\frac{dV_1}{dx_2} = \frac{\lambda}{\mu} x_2 + C_2$$

$$\boxed{V_1 = \frac{\lambda x_2^2}{2\mu} + C_2 x_2 + C_3}$$

2b) B.C. $X=0 \quad V_1=0 \Rightarrow C_3=0$

$X=H \quad V_1=V$

$$V=\frac{\lambda H^2}{\mu 2}+C_2 H$$

$$C_2=\frac{V}{H}-\frac{\lambda H^{\cancel{2}}}{2\mu}\frac{1}{\cancel{H}}$$

$$\boxed{C_2=\frac{V}{H}-\frac{\lambda H}{2\mu}}$$

$$V_1=\frac{\lambda}{2\mu}X_2^2+\frac{V}{H}X_2-\frac{\lambda H}{2\mu}X_2$$

$$=\frac{\lambda}{2\mu}\left(X_2^2-HX_2\right)+\frac{V}{H}X_2$$

$$=\frac{\lambda H^2}{2\mu}\left(\left(\frac{X_2}{H}\right)^2-\left(\frac{X_2}{H}\right)\right)+V\left(\frac{X_2}{H}\right)$$

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$$\boxed{v_1=\left(\frac{P_L-P_0}{2L\mu}\right)H^2\left(\left(\frac{X_2}{H}\right)^2-\left(\frac{X_2}{H}\right)\right)+V\left(\frac{X_2}{H}\right)}$$

6. bonus

$$u_k \underbrace{A_{jk}}_{\substack{\text{but}\\ k\neq j\\ \text{don't}\\ \text{match; must be } \underline{\underline{A}}^T}} \underbrace{\frac{\partial w_p}{\partial x_j} e_p}_{\nabla \underline{w}}$$

try $\underline{u} \cdot \underline{\underline{A}}^T \cdot \nabla \underline{w} =$

$$u_i \hat{e}_i \cdot A_{pk} \hat{e}_k \hat{e}_p \cdot \frac{\partial w_m}{\partial x_s} \hat{e}_s \hat{e}_m$$

δ_{ps} "p becomes s"

δ_{ik} "i becomes k"

$$u_k A_{sk} \frac{\partial w_m}{\partial x_s} \hat{e}_m$$ ✓ correct

$$\boxed{\underline{u} \cdot \underline{\underline{A}}^T \cdot \nabla \underline{w}}$$