# EXAM 2
# CM 4650
# 2 April 08

1. The other components of $\underline{\underline{\tau}}$ are

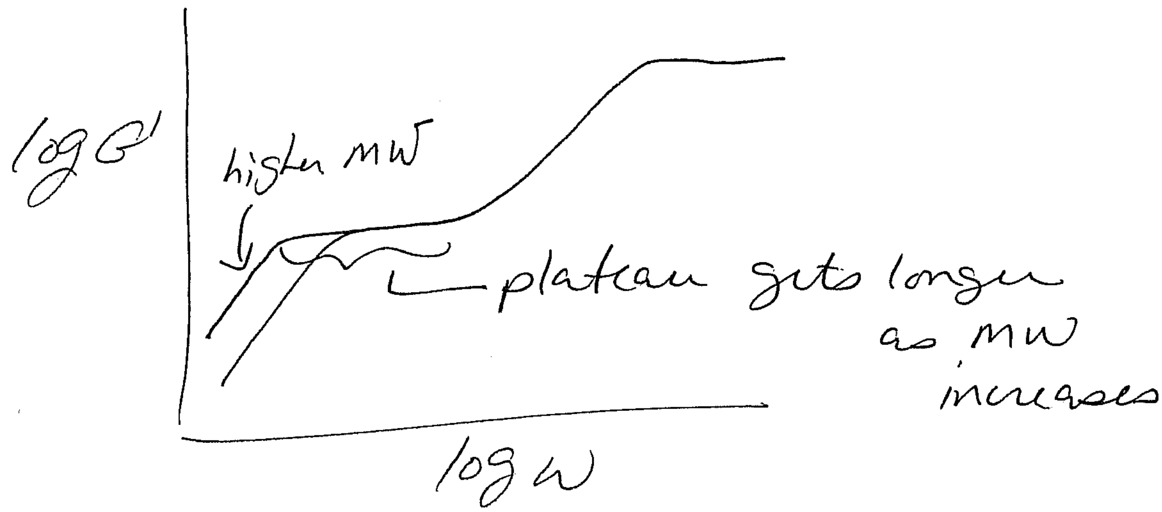
$$\tau_{12} = \tau_{21}$$

$$\left.\begin{array}{l}\tau_{13}\\ \tau_{31}\\ \tau_{23}\\ \tau_{32}\end{array}\right\}$$ these are zero due to symmetry

2. Material function = a defined function of stress based on chosen kinematics (flow). They are used to catalog fluid behavior.

Constitutive Eqn: the full $\underline{\underline{\tau}}$ - $\underline{v}$ relationship. A tensor equation that gives $\underline{\underline{\tau}}$ for any flow.

3.




4. GNF steady shear

$$\underline{\underline{\tau}} = \eta(\dot{\gamma}) \begin{pmatrix} 0 & \dot{\gamma}_0 & 0 \\ \dot{\gamma}_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{123}$$

$$\boxed{\begin{aligned} \tau_{11} - \tau_{22} &= 0 \\ \tau_{22} - \tau_{33} &= 0 \end{aligned}}$$

No, it will not rod-climb since rod climbing is due to normal stress effects + there are no normal stresses predicted.

5. $$\rho \left( \underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\text{steady}} + \underbrace{\cancel{\underline{v} \cdot \nabla \underline{v}}}_{\text{unidir}} \right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \underline{g}$$

$$\underline{v} = \begin{pmatrix} 0 \\ 0 \\ v_z \end{pmatrix}_{xyz} \qquad \underline{g} = \begin{pmatrix} -g \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

$$\nabla \cdot \underline{\underline{\tau}} = \frac{\partial}{\partial x_i} \underbrace{\hat{e}_i \cdot \tau_{mp} \hat{e}_m}_{\delta_{im}} \hat{e}_p$$

"i becomes m"

$$= \frac{\partial \tau_{mp}}{\partial x_m} \hat{e}_p = \begin{pmatrix} \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \end{pmatrix}_{xyz}$$

PL GNF $\Rightarrow$ $\underline{\underline{\tau}} = -\eta \dot{\underline{\underline{\gamma}}}$

$\eta = m \dot{\gamma}^{n-1}$

$$\dot{\underline{\underline{\gamma}}} = \nabla \underline{v} + (\nabla \underline{v})^T$$

$$\nabla \underline{v} = \frac{\partial}{\partial x_i} \hat{e}_i \, v_j \hat{e}_j = \frac{\partial v_j}{\partial x_i} e_i e_j$$

$$\nabla \underline{v} = \begin{pmatrix} \cancel{\frac{\partial v_x}{\partial x}} & \cancel{\frac{\partial v_y}{\partial x}} & \frac{\partial v_z}{\partial x} \\ \cancel{\frac{\partial v_x}{\partial y}} & \cancel{\frac{\partial v_y}{\partial y}} & \frac{\partial v_z}{\partial y} \\ \cancel{\frac{\partial v_x}{\partial z}} & \cancel{\frac{\partial v_y}{\partial z}} & \cancel{\frac{\partial v_z}{\partial z}} \end{pmatrix}_{123}$$

$v_x = 0$ $\quad$ $v_y = 0$

Continuity: $\nabla \cdot \underline{v} = 0 = \cancel{\frac{\partial v_x}{\partial x}} + \cancel{\frac{\partial v_y}{\partial y}} + \frac{\partial v_z}{\partial z}$

$v_x = 0$ $\quad$ $v_y = 0$

$$\Rightarrow \boxed{\frac{\partial v_z}{\partial z} = 0}$$

$$\nabla \underline{v} = \begin{pmatrix} 0 & 0 & \frac{\partial v_z}{\partial x} \\ 0 & 0 & \frac{\partial v_z}{\partial y} \\ 0 & 0 & 0 \end{pmatrix}_{xyz}$$

$$(\nabla \underline{v})^T = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & 0 \end{pmatrix}_{xyz}$$

$$\dot{\underline{\underline{\gamma}}} = \begin{pmatrix} 0 & 0 & \frac{\partial v_z}{\partial x} \\ 0 & 0 & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & 0 \end{pmatrix}_{123}$$

$\nabla$

$\underline{\underline{\tau}} = -\eta \dot{\underline{\underline{\gamma}}} \Rightarrow$ we can simplify $\nabla \cdot \underline{\underline{\tau}}$ with the zeros above.

$$\nabla \cdot \underline{\underline{\tau}} = \underbrace{\begin{pmatrix} \frac{\partial}{\partial z}(\tau_{zx}) \\ \frac{\partial}{\partial z}(\tau_{zy}) \end{pmatrix}}_{} \ldots$$

$$\nabla \cdot \underline{\underline{\tau}} = \begin{pmatrix} \frac{\partial}{\partial z}(\tau_{zx}) \\ \frac{\partial}{\partial z}(\tau_{zy}) \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \end{pmatrix}_{xyz}$$

$v_z$ is not a function of $z$ ∴ these $z$-derivs are zero

$$\nabla \cdot \underline{\underline{\tau}} = \begin{pmatrix} 0 \\ 0 \\ \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right) \end{pmatrix}_{xyz}$$

EOM:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{xyz} = \begin{pmatrix} -\frac{\partial P}{\partial x} \\ -\frac{\partial P}{\partial y} \\ -\frac{\partial P}{\partial z} \end{pmatrix}_{xyz} - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \end{pmatrix}_{xyz} + \begin{pmatrix} -\rho g \\ 0 \\ 0 \end{pmatrix}_{xyz}$$

a

To calculate $\underline{\underline{\tau}}$ we need $\dot{\gamma}$.

$$\underline{\underline{\tau}} = -\eta(\dot{\gamma})\, \underline{\underline{\dot{\gamma}}}$$

$$\dot{\gamma} = |\underline{\underline{\dot{\gamma}}}| = \sqrt{\frac{\underline{\underline{\dot{\gamma}}} : \underline{\underline{\dot{\gamma}}}}{2}}$$

$$\underline{\underline{\dot{\gamma}}} : \underline{\underline{\dot{\gamma}}} = \Bigg( 0 + 0 + \left(\frac{\partial v_z}{\partial x}\right)^2 + 0 + 0$$

$$+ \left(\frac{\partial v_z}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_z}{\partial y}\right)^2 + 0 \Bigg)$$

$$\dot{\gamma} = \sqrt{\frac{2\left[\left(\frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_z}{\partial y}\right)^2\right]}{2}}$$

$$\boxed{\dot{\gamma} = +\sqrt{\left(\frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_z}{\partial y}\right)^2}}$$

(b)

$$\underline{\underline{\tau}} = -\eta \, \underline{\underline{\dot{\gamma}}}$$

$$= -\eta \begin{pmatrix} 0 & 0 & \frac{\partial v_z}{\partial x} \\ 0 & 0 & \frac{\partial v_z}{\partial y} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & 0 \end{pmatrix}$$

$$\eta = m \dot{\gamma}^{\,n-1}$$

$$\dot{\gamma} = \sqrt{\left(\frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_z}{\partial y}\right)^2}$$

(c) Boundary conditions

use 2 out of 3 of each

$x = \pm H$ $\quad V_z = 0$ $\quad$ or $\quad \frac{\partial v_z}{\partial x} = 0$ @ $x = 0$

$y = \pm W$ $\quad V_z = 0$ $\quad$ or $\quad \frac{\partial v_z}{\partial y} = 0$ @ $y = 0$

$z = 0$ $\quad P = P_0$

$z = L$ $\quad P = P_L$