Name:_____

Final Exam

CM 4650

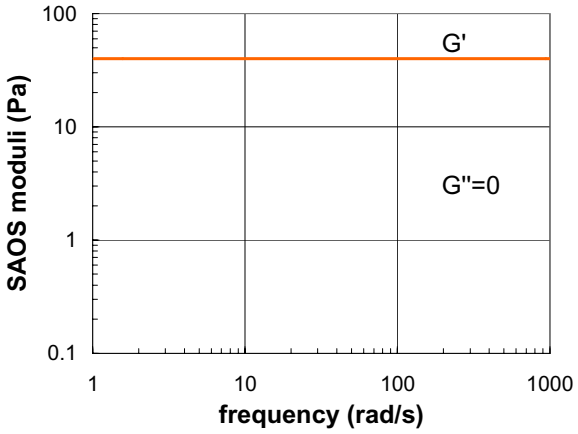
30 April and 2 May 2008

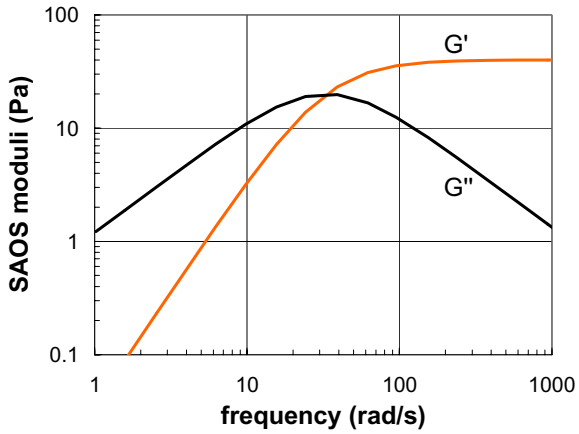
Please be neat.
Please write on only one side of each piece of paper in your solution.
Useful formulas are given on the back page.

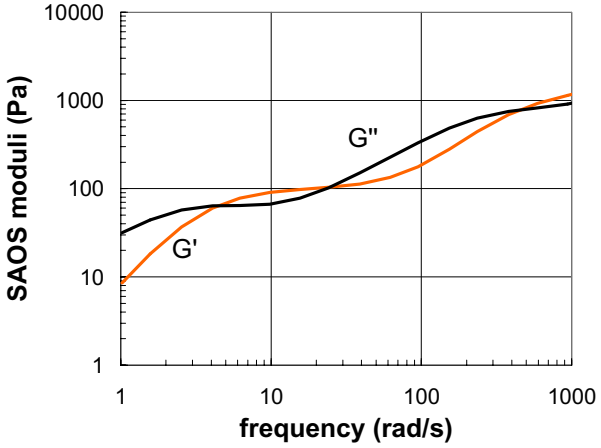
1. (10 points) The parameter λ in the Maxwell model is called the relaxation time. What is the role of the relaxation time in the Maxwell model and in the generalized Maxwell model (i.e. what real behavior does it help to describe?)

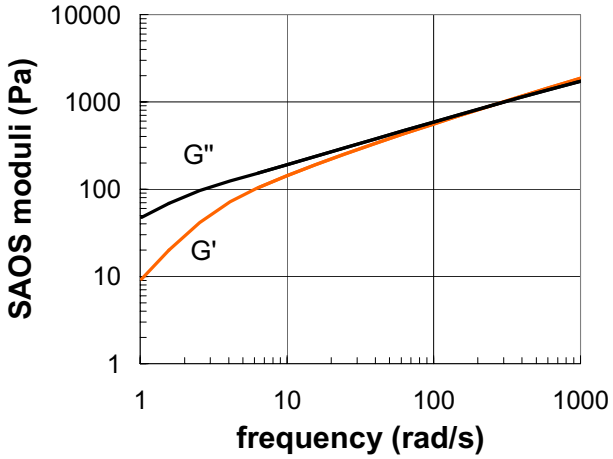
2. (20 points) Write $\nabla \cdot \underline{\underline{\tau}}$ in
   a) Einstein notation and
   b) matrix form

3. (20 points) Three sketches of $G'(\omega)$ and $G''(\omega)$ are shown below. Which one is a) a single relaxation-time Maxwell model; b) the Rouse model; and c) an elastic solid. You may write the name on the graph.

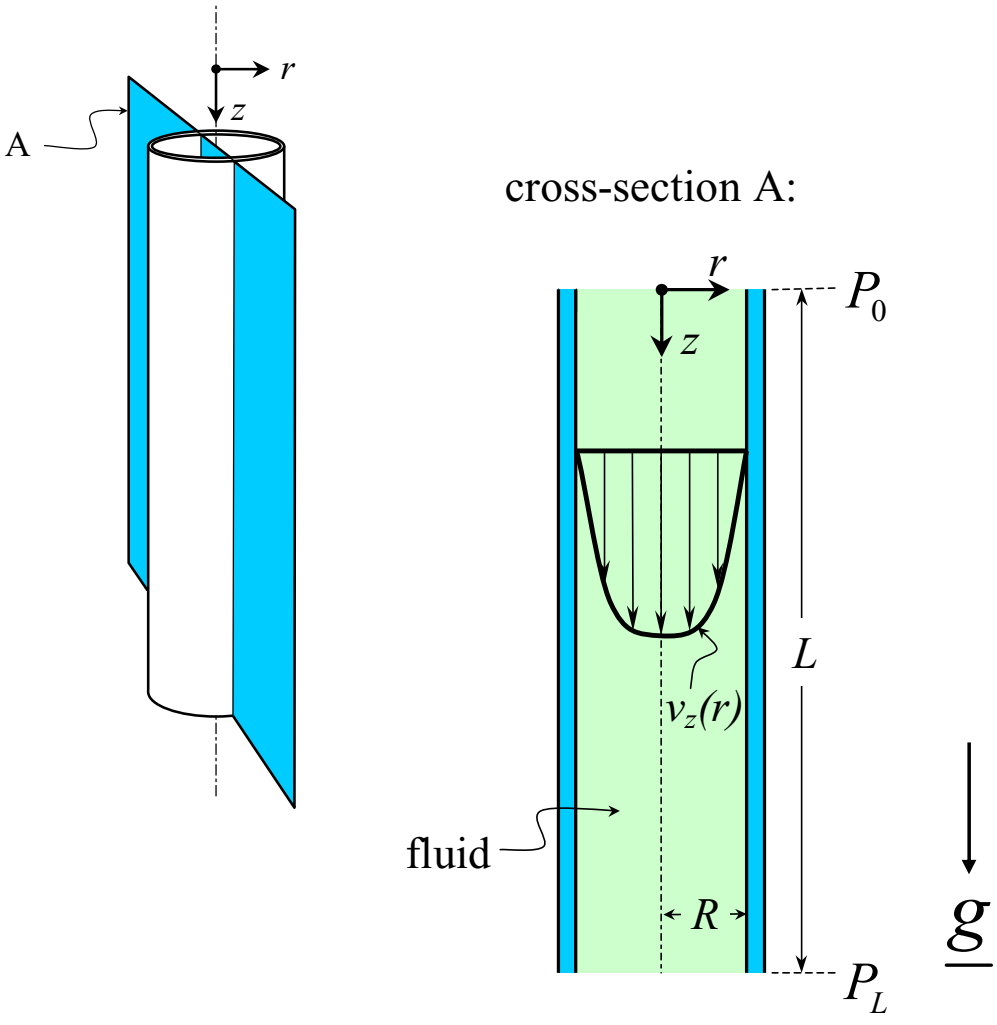








4. (20 points) An incompressible, power-law, generalized Newtonian fluid is made to flow in a tube of circular cross-section (see figure below). The flow is steady. The upstream pressure at $z=0$ is P_0, and the pressure a distance L downstream is P_L. What is the differential equation for velocity as a function of position? What are the boundary conditions?



5. (30 points) For the material function $\bar{\eta}(\dot{\varepsilon}_0)$, the steady shear elongational viscosity as a function of elongation rate, the Lodge model makes the prediction shown below. Show that this is so. You must show your work for full points.

$$\bar{\eta}(\dot{\varepsilon}_0) = \frac{3\eta_0}{(1+\dot{\varepsilon}_0\lambda)(1-2\dot{\varepsilon}_0\lambda)}$$

6. Bonus (10 points): The Finger tensor for shear is given by the expression below with the strain γ defined as

$$\gamma = \gamma(t',t) = \int_{t'}^{t} \dot{\varsigma}(t'')dt''$$

$$\underline{\underline{C}}^{-1}(t',t) = \begin{pmatrix} 1+\gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{123}$$

For the step-strain material function, the kinematic function $\dot{\varsigma}(t'')$ is given by

$$\dot{\varsigma}(t'') = \begin{cases} 0 & t'' < 0 \\ \dfrac{\gamma_0}{\varepsilon} & 0 < t'' < \varepsilon \\ 0 & \varepsilon < t'' \end{cases}$$

where γ_0 and ε are constants, and all calculations are evaluated in the limit that ε goes to zero.

What is the Finger tensor strain function $\gamma = \gamma(t',t)$ for step shear strain? Note that you will want to consider three regions of t', $t' < 0$, $0 < t' < \varepsilon$, and $\varepsilon < t'$.