Name:________________________

Final Exam

CM 4650

28 April 2009

Please be neat.
Please write on only one side of each piece of paper in your solution.
Useful formulas are given on a separate page.

1. (20 points) Please answer the questions below. Please be brief.

   a) (5 points) Give one example of a material function (give the name and the symbol)

   b) (5 points) Give one example of a constitutive equation (give the name and the equation).

   c) (5 points) From the list below, circle all the models that are capable of predicting polymeric memory effects in shear flow (see attached sheet for model equations):

      Newtonian fluid model

      Power-law generalized Newtonian fluid model

      Maxwell model

      Generalized Linear Viscoelastic fluid model

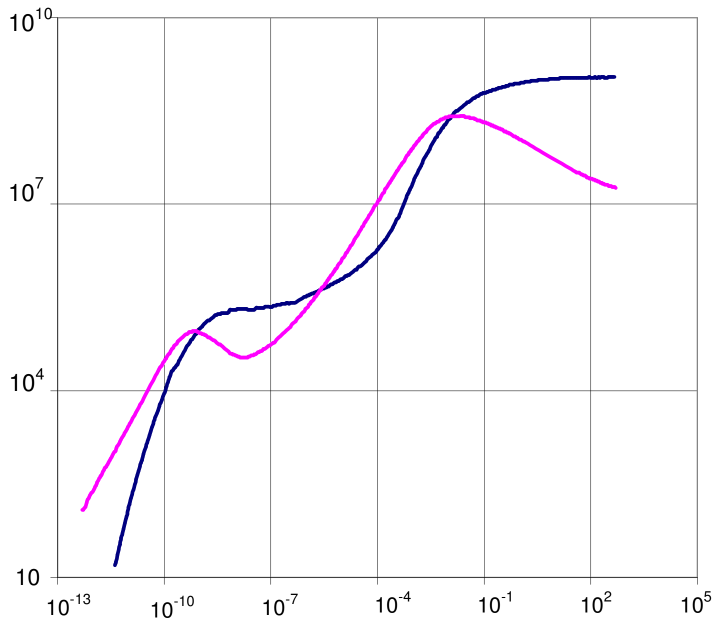
      Lodge model

   d) (5 points) True or False? The capillary rheometer is capable of measuring viscosity as a function of shear rate for polymer melts.

2. (20 points) In terms of the Cartesian components of $\underline{\underline{A}}$ and $\underline{u}$, what is the 23-component of the expression $\nabla\left(\underline{\underline{A}} \cdot \underline{u}\right)$? Derive using Einstein notation and please include any summation signs that are required; carry out the product rule of differentiation.

3. (10 points) The small-amplitude oscillatory shear functions G' and G" for a polymer melt are plotted below. Place the following labels on the graph:

   a) Label one curve G' and the other curve G"
   b) Label the *x*-axis (what is the variable, and what would be reasonable units?)
   c) Label the *y*-axis with reasonable units
   d) Indicate the plateau modulus
   e) Indicate which region is the "terminal zone"



4. (25 points)

   a) Calculate steady shear viscosity η for the constitutive equation given below; *g*, λ, and ϕ_1 are constant parameters of the model. Please express your final answer as a simple integral over $s = t - t'$.

   b) Does this model predict a nonzero Ψ_2 in steady shear flow? Justify your answer, but you do not need to calculate Ψ_2.

   c) **BONUS** (5 points): carry out the integral from part a) and obtain the final prediction for the steady shear viscosity η.

$$\underline{\underline{\tau}}(t) = -\int_{-\infty}^{t} \frac{g}{\lambda} e^{-\frac{t-t'}{\lambda}} \left[\phi_1 \underline{\underline{C}}^{-1}(t',t) + \underline{\underline{C}}(t,t') \right] dt'$$

5. (25points) Electrical wire (circular cross-section) can be coated with insulation by drawing the wire at a velocity V through a cylindrical liquid bath containing the coating fluid. A schematic of this operation is shown below. Away from the ends, the flow in the bath is a drag flow with cylindrical symmetry, *and pressure is constant throughout*.

Calculate the *velocity profile* in the fluid at steady state for the center portion of the flow (away from the ends; L is long compared to R). The liquid in the bath is an incompressible, power-law, generalized Newtonian fluid. You may neglect gravity. You may leave your answer in terms of the integration constants; be sure to indicate the boundary conditions.

cross-section A: