Homework 2
CM4650
Spring 2009

Due: *Wednesday February 4, 2009, in class*

Please write on one side of the page only! Please write legibly and large. Thank you.

1. (10 points) For the three vectors given below, what are the following quantities equal to?
   a. the gradient of the vector ($\nabla(vector)$)
   b. the divergence of the vector ($\nabla\cdot(vector)$)
   c. the Laplacian of the vector ($\nabla^2(vector)=\nabla\cdot\nabla(vector)$)

$$\underline{a}=\begin{pmatrix}4x+y\\2x\\-0.8x-2z\end{pmatrix}_{xyz}$$

$$\underline{b}=x^2\hat{e}_x+2y^2\hat{e}_y+2x\hat{e}_z$$

$$\underline{v}=(x+z)\hat{e}_x-2xe^x\hat{e}_y+\sin x\hat{e}_z$$

2. (10 points) Work out the gradient $\nabla(\underline{a}\cdot\underline{v})$ in Einstein notation for general vectors $\underline{a}$ and $\underline{v}$. (carry out the product rule). What is $\nabla(\underline{a}\cdot\underline{v})$ equal to for the vectors given below? Verify that calculating the gradient as $\nabla(\underline{a}\cdot\underline{v})$ or calculating it using your answer to the first part both give the same answer.

$$\underline{a}=\begin{pmatrix}3x^2+y\\2y\\1\end{pmatrix}_{xyz}$$

$$\underline{v}=x\hat{e}_x+2z\hat{e}_y-\hat{e}_z$$

3. (10 points) What is $a_p m_g Y_{gj}\hat{e}_j\hat{e}_p$ in Gibbs (vector-tensor) notation?
4. (10 points) What is the magnitude of the tensor $\nabla\underline{a}$ for $\underline{a}$ given in problem 1?
5. (20 points) Text 3.16 (combined pressure and drag of a Newtonian fluid in a slit)

OVER

6. (30 points) (lengthy!) An incompressible, Newtonian fluid is confined in the gap between two concentric cylinders. The outer cylinder is turned at a constant angular speed Ω. The outer cylinder has a radius R and the inner cylinder has a radius κR and the gap between the two cylinders is small. What is the velocity profile in the gap?