Name:_____

Exam I

CM 4650

February 20, 2007

Please be neat.

Please write on only one side of each piece of paper in your solution.

Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\mu\nabla^2\underline{v}+\rho\underline{g}$

Continuity Equation: $\frac{\partial\rho}{\partial t}=-\nabla\cdot(\rho\underline{v})$

Newtonian Incompressible Constitutive Equation: $\underline{\underline{\tau}}=-\mu\left[\nabla\underline{v}+(\nabla\underline{v})^T\right]$

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1. (10 points) Are $\begin{pmatrix}1\\2\\-1\end{pmatrix}_{123}$ and $\begin{pmatrix}-1\\0\\1\end{pmatrix}_{123}$ perpendicular? Why or why not?

2. (20 points) What is $\left(\underline{v}\cdot\underline{\underline{B}}^T\right)$ written in Einstein notation?

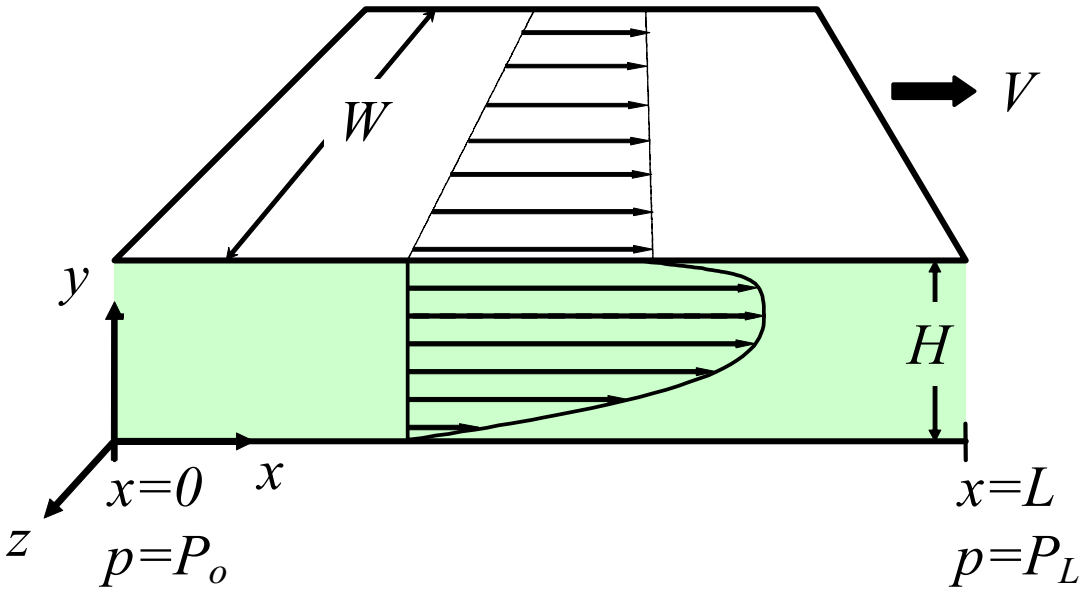
3. (20 points) For the scalar function *f* written below, what is $\nabla\cdot\nabla f$?

$$f(x_1,x_2,x_3)=3x_1^2-2x_2+x_3$$

4. (20 points) What is the 21-component of $\underline{\underline{A}}=(\nabla\underline{w})^T+\underline{\underline{B}}$

5. (30 points) A steady flow of an incompressible, Newtonian fluid is created between two very wide, parallel plates as shown below. The pressure at position $x=0$ is P_0 and the pressure at position $x=L$ is P_L. The top plate moves with a steady velocity V. Answer the questions below. You may neglect gravity. Please show your work.

    1. What is the differential equation for the velocity, v_x and the pressure, p?
    2. What are the boundary conditions on velocity and pressure? (make sure you give enough boundary conditions to evaluate all the integration constants)
    3. What is the steady state pressure profile?
    4. What is the steady state velocity profile?



6. (5 point bonus problem) For the tensor given below, the first invariant $I_{\underline{\underline{\dot{\gamma}}}} = trace(\underline{\underline{\dot{\gamma}}}) = 0$. What is the first invariant equal to in the cylindrical coordinate system?

$$\underline{\underline{\dot{\gamma}}} = \begin{pmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 8 \end{pmatrix}_{123}$$