Name:_____

Exam I

CM 4650

February 14, 2008

Please be neat.
Please write on only one side of each piece of paper in your solution.
This exam is closed book, closed notes.

Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\mu\nabla^2\underline{v}+\rho\underline{g}$

Continuity Equation: $\frac{\partial\rho}{\partial t}=-\nabla\cdot(\rho\underline{v})$

Newtonian Incompressible Constitutive Equation: $\underline{\underline{\tau}}=-\mu\left[\nabla\underline{v}+(\nabla\underline{v})^T\right]$

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1. (10 points) In the Navier-Stokes equation below, circle the term or terms that account for the convection of momentum into and out of the control volume.

   Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t}+\underline{v}\cdot\nabla\underline{v}\right)=-\nabla p+\mu\nabla^2\underline{v}+\rho\underline{g}$

2. (20 points) What is $(\underline{a}\cdot\nabla\underline{b})$ written in Einstein notation?

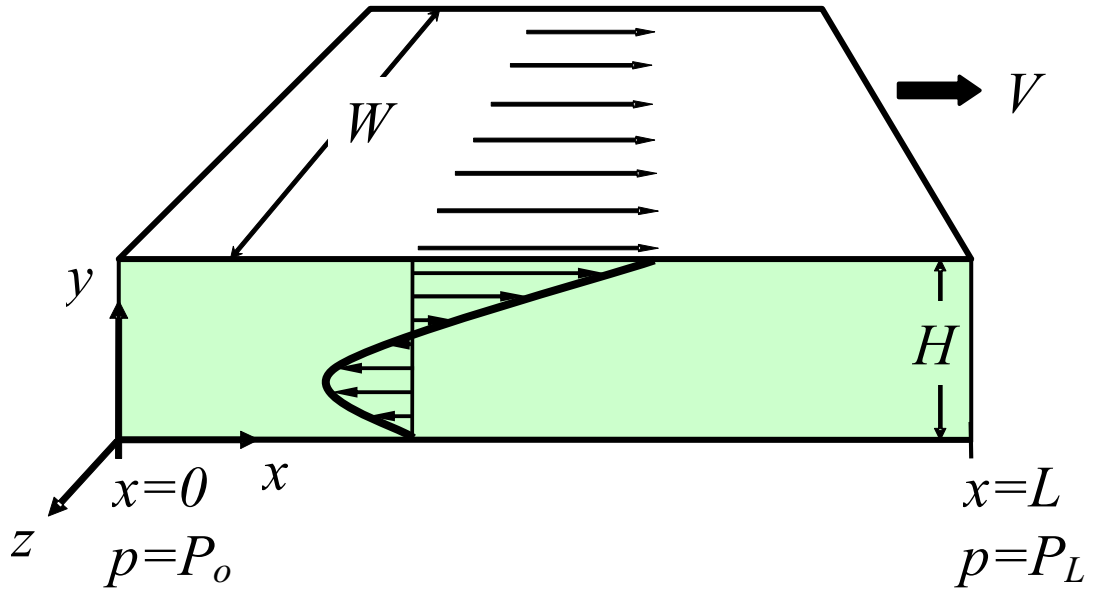
3. (20 points) Supply the missing summations in the expression below. Please be complete.

   $$u_k A_{jk}\frac{\partial w_p}{\partial x_j}\hat{e}_p$$

4. (20 points) What is the 21-component of $\underline{\underline{A}}=\nabla\underline{v}+(\nabla\underline{v})^T$?

5. (30 points) A steady flow of an incompressible, Newtonian fluid is created between two very wide, parallel plates as shown below. The pressure at position $x=0$ is P_0, and the pressure at position $x=L$ is P_L. The top plate moves with a steady velocity V, while the bottom plate is stationary. P_L is greater than P_0 causing a backward flow near the bottom of the two plates. Answer the questions below. You may neglect gravity. Please show your work and indicate your assumptions clearly.

    1. What is the differential equation for the velocity, v_x and the pressure, p?
    2. What are the boundary conditions on velocity and pressure? (make sure you give enough boundary conditions to evaluate all the integration constants)
    3. What is the steady state pressure profile?
    4. What is the steady state velocity profile?



6. (BONUS 5 points) Write the expression in problem 3 in Gibbs (vector/tensor) notation.