EXAM 1
CM4650
SOLN

1. $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = -1 + 0 + -1 = -2 \neq 0$

No, they are not $\perp$ because
$\underline{v} \cdot \underline{w} \neq 0$ //

2. $$\underline{v} \cdot \underline{\underline{B}}^T = v_p \hat{e}_p \cdot \left(B_{jm} \hat{e}_j \hat{e}_m\right)^T$$

$$= v_p \underbrace{\hat{e}_p \cdot B_{jm} \hat{e}_m}_{\delta_{pm}} \hat{e}_j$$

"p becomes m"

$$= v_m B_{jm} \hat{e}_j$$ //

(3) $\nabla \cdot \nabla f = \underbrace{\frac{\partial}{\partial x_m} \hat{e}_m \cdot \frac{\partial}{\partial x_p} \hat{e}_p}_{\delta_{mp}} f$ "m becomes p"

$$= \frac{\partial}{\partial x_p} \frac{\partial}{\partial x_p} f$$

$$= \frac{\partial^2 f}{\partial x_p} = \frac{\partial^2 f}{\partial x_1^2} + \frac{\partial^2 f}{\partial x_2^2} + \frac{\partial^2 f}{\partial x_3^2}$$

$\frac{\partial f}{\partial x_1} = 2(3x_1)$ $\quad \frac{\partial^2 f}{\partial x_1^2} = 6$

$\frac{\partial f}{\partial x_2} = -2$ $\quad \frac{\partial^2 f}{\partial x_2^2} = 0$

$\frac{\partial f}{\partial x_3} = 1$ $\quad \frac{\partial^2 f}{\partial x_3} = 0$

$$\nabla \cdot \nabla f = 6 + 0 + 0 = \boxed{6}$$

4. $\underline{\underline{A}} = (\nabla \underline{w})^T + \underline{\underline{B}}$

$$= \left(\frac{\partial}{\partial x_s}\hat{e}_s \; w_j \hat{e}_j\right)^T + B_{mf}\,\hat{e}_m \hat{e}_f$$

$$= \frac{\partial w_j}{\partial x_s}\,\underset{\underset{2}{\shortparallel}}{\hat{e}_j}\,\underset{\underset{1}{\shortparallel}}{\hat{e}_s} + B_{mf}\,\underset{\underset{2}{\shortparallel}}{\hat{e}_m}\,\underset{\underset{1}{\shortparallel}}{\hat{e}_f}$$

$$\boxed{A_{21} = \frac{\partial w_2}{\partial x_1} + B_{21}}$$

//

continuity: 5-1

5. $\nabla \cdot \underline{V} = 0 = \frac{\partial}{\partial x_i} \hat{e}_i \cdot v_p \hat{e}_p$

δ_{ip} "i becomes p"

$= \frac{\partial v_p}{\partial x_p} = \frac{\partial v_1}{\partial x_1} + \cancel{\frac{\partial v_2}{\partial x_2}} + \cancel{\frac{\partial v_3}{\partial x_3}} = 0$

$\underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \boxed{\frac{\partial v_1}{\partial x_1} = 0}$

E.O.M:

$\rho\left(\cancel{\frac{\partial \underline{v}}{\partial t}} + \cancel{\underline{v} \cdot \nabla \underline{v}}\right) = -\nabla p + \mu \nabla^2 \underline{v} + \cancel{\rho \underline{g}}$

steady unidir $\frac{\partial p}{\partial x_i} \hat{e}_i$ neglect

$\nabla^2 \underline{v} = \frac{\partial}{\partial x_i} \hat{e}_i \cdot \frac{\partial}{\partial x_p} \hat{e}_p v_m \hat{e}_m$

δ_{ip} "i becomes p"

$= \frac{\partial}{\partial x_p} \frac{\partial}{\partial x_p} v_m \hat{e}_m$

$$\nabla^2 \underline{v} = \begin{pmatrix} \sum_{p=1}^{3} \frac{\partial^2 v_1}{\partial x_p^2} \\ \cancelto{0}{\sum_{p=1}^{3} \frac{\partial^2 v_2}{\partial x_p^2}} \\ \cancelto{0}{\sum_{p=1}^{3} \frac{\partial^2 v_3}{\partial x_p^2}} \end{pmatrix}_{123}$$

continuity

$$\nabla^2 \underline{v} = \begin{pmatrix} \cancel{\frac{\partial^2 v_1}{\partial x_1^2}} + \frac{\partial^2 v_1}{\partial x_2^2} + \cancel{\frac{\partial^2 v_1}{\partial x_3^2}} \\ 0 \\ 0 \end{pmatrix}_{123}$$ wide plates

EOM:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{123} = \begin{pmatrix} -\frac{\partial P}{\partial x_1} \\ -\frac{\partial P}{\partial x_2} \\ -\frac{\partial P}{\partial x_3} \end{pmatrix}_{123} + \begin{pmatrix} \mu \frac{\partial^2 v_1}{\partial x_2^2} \\ 0 \\ 0 \end{pmatrix}$$

X_2-component $\boxed{\dfrac{\partial P}{\partial X_2} = 0}$

X_3-component $\boxed{\dfrac{\partial P}{\partial X_3} = 0}$

X_1-component $\boxed{\underbrace{\dfrac{\partial P}{\partial X_1}}_{\text{function of } X_1 \text{ only}} = \underbrace{\mu \dfrac{\partial^2 U_1}{\partial X_2^2}}_{\text{function of } X_2 \text{ only}}} = \lambda$

$$\frac{dP}{dX_1} = \lambda$$

$$\boxed{P = \lambda X_1 + C_1}$$

BC: $X_1 = 0 \quad P = P_0 \Rightarrow \boxed{C_1 = P_0}$

$X_1 = L \quad P = P_L$

$$\Rightarrow P_L = \lambda L + P_0$$

$$\boxed{\lambda = \frac{P_L - P_0}{L}}$$

$$\boxed{P = \frac{P_L - P_0}{L} X_1 + P_0}$$

$$\frac{d^2V_1}{dX_2^2} = \frac{1}{\mu}\lambda = \frac{P_L - P_0}{\mu L}$$

$$\frac{dV_1}{dX_2} = \frac{P_L - P_0}{\mu L} X_2 + C_3$$

$$\boxed{V_1 = \frac{P_L - P_0}{2\mu L} X_2^2 + C_3 X_2 + C_4}$$

B.C. $X_2 = 0 \quad V_1 = 0 \Rightarrow \boxed{C_4 = 0}$

$X_2 = H \quad V_1 = V$

$$\Rightarrow V = \frac{P_L - P_0}{2\mu L} H^2 + C_3 H$$

$$\boxed{C_3 = \frac{V - \frac{(P_L - P_0)H^2}{2\mu L}}{H}}$$

$$V_1 = \left(\frac{P_L - P_0}{2\mu L}\right) X_2^2 + \frac{X_2}{H}\left(V - \left(\frac{P_L - P_0}{2\mu L}\right) H^2\right)$$

$$V_1 = \left(\frac{P_L - P_0}{2\mu L}\right)\left[X_2^2 - X_2 H\right] + \frac{X_2}{H} V$$

$$\boxed{V_1 = \left(\frac{P_L - P_0}{2\mu L}\right) H^2 \left[\left(\frac{X_2}{H}\right)^2 - \left(\frac{X_2}{H}\right)\right] + \frac{X_2}{H} V}$$

Alternate BC: $X_2 = \beta \quad \frac{dv_1}{dX_2} = 0$

$$\Rightarrow \boxed{C_3 = -\left(\frac{P_L - P_0}{\mu L}\right)\beta}$$

$$\boxed{V_1 = \left(\frac{P_L - P_0}{2\mu L}\right)\left[X_2^2 - 2\beta X_2\right]}$$