Name:________________________

Exam II

CM 4650

April 3, 2007

Please be neat.
Please write on only one side of each piece of paper in your solution.
Useful formulas are given on the back page.

1. (10 points) Stress is a tensor. The stress constitutive equation is stress written as a function of the fluid motion, for example stress as a function of velocity field $\underline{\underline{\tau}}(\underline{v})$. We have said that the constitutive equation may only be a function of vectors and tensors such as $\underline{v}$ or $\underline{\underline{\dot{\gamma}}}$, or of the scalar invariants of vectors and tensors. Why are stress constitutive equations limited to being a function of invariants?

2. (15 points) Please answer the following.

   a) For a typical long-chain, entangled polymer, please sketch the shear viscosity and first-normal-stress coefficient as a function of shear rate (log-log graph). Please be sure to label your graph.

   b) When a filler (for example a dry powder like chalk or carbon black) is added to a long-chain, entangled polymer, what is the effect on the viscosity curve? Please sketch your answer and please be sure to label your graph.

   c) For a Newtonian fluid of viscosity μ, what is the elongational viscosity $\bar{\eta}$ equal to?

3. (20 points) The generalized Newtonian fluid constitutive equation is given below:

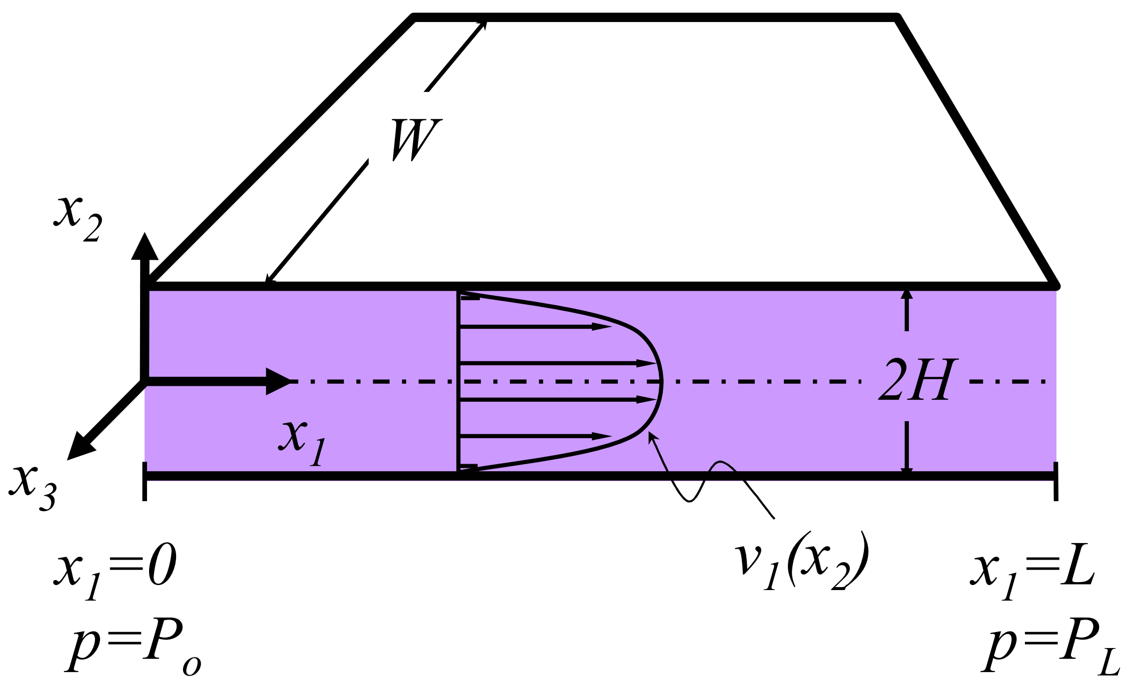
$$\underline{\underline{\tau}} = -\eta(\dot{\gamma})\left[\nabla \underline{v} + (\nabla \underline{v})^T\right] \quad , \quad \eta(\dot{\gamma}) = m\dot{\gamma}^{n-1} \, , \; \dot{\gamma} = \left|\underline{\underline{\dot{\gamma}}}\right|$$

Does this constitutive equation predict memory effects? Why or why not?

4. (25 points) Calculate the steady uniaxial elongational viscosity $\bar{\eta}(\dot{\varepsilon}_0)$ for a power-law, generalized Newtonian fluid. The material function $\bar{\eta}(\dot{\varepsilon}_0)$ is defined as follows: uniaxial elongation, $\dot{\varepsilon}(t) = \dot{\varepsilon}_0 = \text{constant}$; $\dot{\varepsilon}_0 > 0$; $\bar{\eta}(\dot{\varepsilon}_0) \equiv \dfrac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$.

5. (30 points) A steady flow of an incompressible, power-law, generalized Newtonian fluid is created between two very wide, parallel plates as shown below. The pressure at position $x_1=0$ is P_0 and the pressure at position $x_1=L$ is P_L. Answer the questions below. You may neglect gravity. Please show your work.

    1. What are the differential equations for the velocity, v_1 and the pressure, p? What are the boundary conditions on velocity and pressure?
    2. What is the steady state pressure profile?
    3. What is the steady state velocity profile? Your answer may include integration constants.
    4. What is the magnitude of the force on the top plate?



Useful Formulas

Magnitude of a tensor $\underline{\underline{A}}$: $\left|\underline{\underline{A}}\right| \equiv +\sqrt{\dfrac{\underline{\underline{A}}:\underline{\underline{A}}}{2}}$

Navier-Stokes Equation: $\rho\left(\dfrac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$

Equation of Motion: $\rho\left(\dfrac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$

Continuity Equation: $\dfrac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\underline{v})$

Newtonian Constitutive Equation: $\underline{\underline{\tau}} = -\mu\left[\nabla\underline{v} + (\nabla\underline{v})^T\right]$

Power-Law Generalized Newtonian Constitutive Equation:

$$\underline{\underline{\tau}} = -\eta(\dot{\gamma})\left[\nabla\underline{v} + (\nabla\underline{v})^T\right] \quad , \quad \eta(\dot{\gamma}) = m\dot{\gamma}^{n-1}$$

Steady shear flow:

$$\underline{v} = \begin{pmatrix} \varsigma(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \qquad \varsigma(t) = \dot{\gamma}_0$$

Steady uniaxial elongational flow:

$$\underline{v} = \begin{pmatrix} -\dot{\varepsilon}(t)x_1/2 \\ -\dot{\varepsilon}(t)x_2/2 \\ \dot{\varepsilon}(t)x_3 \end{pmatrix}_{123} \qquad \dot{\varepsilon}(t) = \dot{\varepsilon}_0$$