Name:________________________

Final Exam

CM 4650

May 2, 2007

Please be neat.

Please write on only one side of each piece of paper in your solution.

Useful formulas are given on the accompanying page.

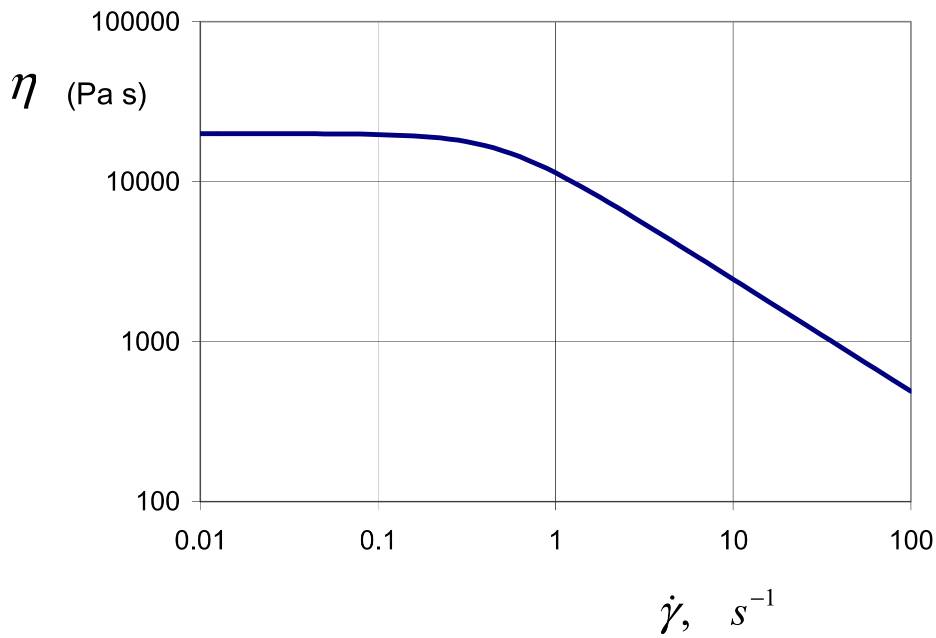
1. (10 points) Please answer true or false for the following statements.
   a) The capillary rheometer is good for measuring steady shear viscosity at high shear rates.
   b) To obtain steady shear viscosity as a function of shear rate using the cone and plate rheometer we must apply a complicated correction procedure.
   c) Elongational viscosity measurements are challenging to carry out because the required deformation is severe.
   d) In a capillary rheometer slip, at the walls can be a problem for some materials.

2. (20 points) Is $\nabla(a\underline{v}) = a\nabla\underline{v} + (\nabla a)\underline{v}$ or is it equal to $\nabla(a\underline{v}) = a(\nabla\underline{v})^T + \underline{v}(\nabla a)$? Note that a is a scalar variable, not a constant. Prove your answer using Einstein notation.

3. (20 points)

a) How does zero-shear viscosity η_0 vary with molecular weight M? Please answer with an equation or with a sketch.

b) In the figure below is a sketch of viscosity as a function of shear rate for a polymer. On the same axes, sketch the viscosity as a function of shear rate for a polymer of higher molecular weight.



4. (20 points) What is the cessation of steady shearing material function $\eta^{-}(t)$ for the Lodge model? Please give your answer in terms of an integral over *t'* or *s=t-t'*; there is no need to carry out the integration.

   Cessation of steady shearing kinematics and material function definition:

$$\underline{v} = \begin{pmatrix} \dot{\varsigma}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \quad , \quad \dot{\varsigma}(t) = \begin{cases} \dot{\gamma}_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$$

$$\eta^{-}(t) \equiv \frac{-\tau_{21}(t)}{\dot{\gamma}_0}$$

   BONUS: (5 points) The Lodge model was developed as a modification of the Maxwell Model. What did we do to the Maxwell model to arrive at the Lodge model? Why was this important?

5. (30 points) A steady flow of an incompressible, power-law, generalized Newtonian fluid,

$$\underline{\underline{\tau}} = -m\dot{\gamma}^{n-1}\underline{\underline{\dot{\gamma}}} = -m\dot{\gamma}^{n-1}\left(\nabla\underline{v} + (\nabla\underline{v})^T\right), \quad \dot{\gamma} = \left|\underline{\underline{\dot{\gamma}}}\right|$$

is created between two very wide, parallel plates as shown below. The pressure at $x_1=0$ is P_0 and the pressure at $x_1=L$ is P_L. Note that P_L (the downstream pressure) is larger than the upstream pressure P_0, but the pressure difference is not enough to reverse the flow direction at any point. The top plate moves with steady velocity V. Gravity may be neglected. Answer the questions below and please show your work.

a) What are the differential equations for v_1 and p?
b) What are the boundary conditions for velocity and pressure?
c) What is the solution for pressure as a function of position?
d) What is the solution for velocity as a function of position? You may leave the integration constants in your solution for v_1 (no need to calculate them).