Final Exam Formulas

Polymer Rheology
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Rate of deformation: $\dot{\gamma} = \left|\underline{\underline{\dot{\gamma}}}\right|$

Tensor magnitude: $A = \left|\underline{\underline{A}}\right| = +\sqrt{\frac{\underline{\underline{A}} : \underline{\underline{A}}}{2}}$

Navier-Stokes Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p + \mu\nabla^2\underline{v} + \rho\underline{g}$

Momentum Equation: $\rho\left(\frac{\partial \underline{v}}{\partial t} + \underline{v}\cdot\nabla\underline{v}\right) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\underline{g}$

Continuity Equation: $\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\underline{v})$

Newtonian, incompressible fluid: $\underline{\underline{\tau}} = -\mu\left(\nabla\underline{v} + (\nabla\underline{v})^T\right)$

Generalized Newtonian fluid (GNF): $\underline{\underline{\tau}} = -\eta(\dot{\gamma})\underline{\underline{\dot{\gamma}}}$

Power-law GNF model: $\eta(\dot{\gamma}) = m\dot{\gamma}^{n-1}$

(Note that m and n are parameters of the model and are constants)

Carreau-Yasuda GNF model: $\eta(\dot{\gamma}) = \eta_\infty + (\eta_o - \eta_\infty)\left[1 + (\dot{\gamma}\lambda)^a\right]^{\frac{n-1}{a}}$

(Note that a, λ and n, η_o, and η_∞ are parameters of the model and are constants)

Generalized Linear Viscoelastic Fluid model (GLVE): $\underline{\underline{\tau}} = -\int_{-\infty}^{t} G(t-t')\,\underline{\underline{\dot{\gamma}}}(t')dt'$

Maxwell model (differential version): $\underline{\underline{\tau}} + \lambda\frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0\underline{\underline{\dot{\gamma}}}$

Maxwell model (integral version): $\underline{\underline{\tau}} = -\int_{-\infty}^{t}\left[\frac{\eta_0}{\lambda}e^{-\frac{(t-t')}{\lambda}}\right]\underline{\underline{\dot{\gamma}}}(t')dt'$

Generalized Maxwell model (GMM): $\underline{\underline{\tau}} = -\int_{-\infty}^{t}\left[\sum_{k=1}^{N}\frac{\eta_k}{\lambda_k}e^{-\frac{(t-t')}{\lambda_k}}\right]\underline{\underline{\dot{\gamma}}}(t')dt'$

Lodge model: $\underline{\underline{\tau}} = -\int_{-\infty}^{t}\left[\frac{\eta_0}{\lambda^2}e^{-\frac{(t-t')}{\lambda}}\right]\underline{\underline{C}}^{-1}(t',t)dt'$

Elongational flow (uniaxial, biaxial): $\underline{v} = \begin{pmatrix} -\dfrac{\varepsilon(t)}{2} x_1 \\ -\dfrac{\varepsilon(t)}{2} x_2 \\ \varepsilon(t) x_3 \end{pmatrix}_{123}$

Shear flow: $\underline{v} = \begin{pmatrix} \varsigma(t)\, x_2 \\ 0 \\ 0 \end{pmatrix}_{123}$

Steady shearing kinematics: $\dot{\varsigma}(t) = \dot{\gamma}_0$

Start-up of steady shearing kinematics: $\varsigma(t) = \begin{cases} 0 & t < 0 \\ \gamma_0 & t \geq 0 \end{cases}$

Cessation of steady shearing kinematics: $\varsigma(t) = \begin{cases} \gamma_0 & t < 0 \\ 0 & t \geq 0 \end{cases}$

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Shear viscosity: $\eta = \dfrac{-(\tau_{21})}{\dot{\gamma}_0}$

Shear normal stress coefficients: $\Psi_1 = \dfrac{-(\tau_{11} - \tau_{22})}{\dot{\gamma}_0^2}, \Psi_2 = \dfrac{-(\tau_{22} - \tau_{33})}{\dot{\gamma}_0^2}$

Elongational viscosity: $\bar{\eta} = \dfrac{-(\tau_{33} - \tau_{11})}{\dot{\varepsilon}_0}$

Also: Table 9.3 (p 329)