

Predictions of the (single-mode) Maxwell Model

$$\underline{\underline{\tau}} + \frac{\eta_0}{G} \frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0 \underline{\underline{\gamma}}$$

$$\underline{\underline{\tau}}(t) = - \int_{-\infty}^t \left(\frac{\eta_0}{\lambda} \right) e^{-(t-t')/\lambda} \underline{\underline{\gamma}}(t') dt'$$

Steady shear

$$\eta = \eta_0$$

Fails to predict shear normal stresses.

$$\Psi_1 = \Psi_2 = 0$$

Fails to predict shear-thinning.

Steady elongation

$$\bar{\eta} = 3\eta_0$$

Trouton's rule

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Steady shear viscosity and first normal stress coefficient

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There are some systems with a constant viscosity but still start-up effects.

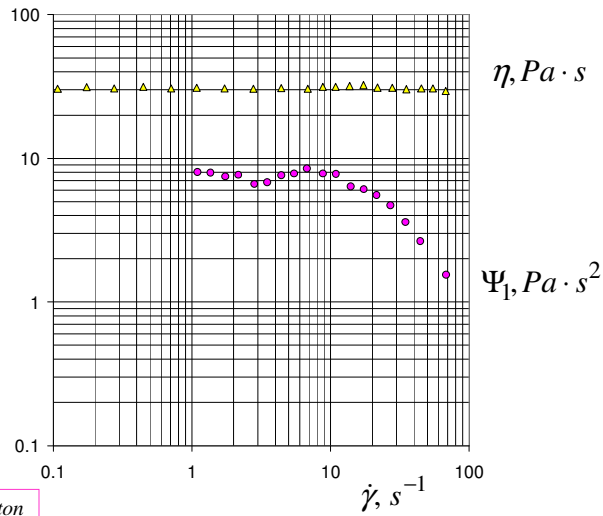


Figure 6.5, p. 173 Binnington and Boger; PIB soln

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Steady shear viscosity and first and *second* normal stress coefficient

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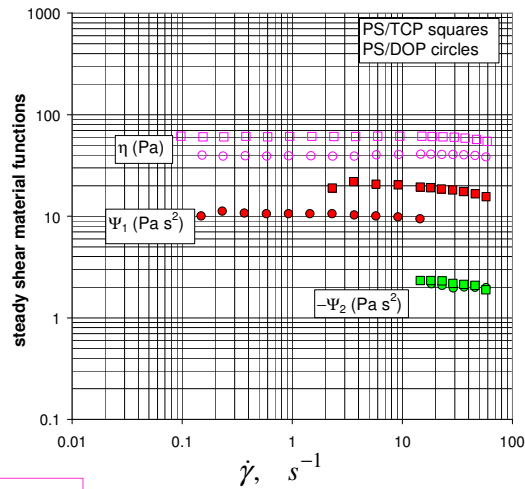


Figure 6.6, p. 174 Magda et al.; PS solns

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Step Shear Strain Material Functions

Kinematics:

$$\underline{v} \equiv \begin{pmatrix} \dot{\zeta}(t)x_2 \\ 0 \\ 0 \end{pmatrix}_{123} \quad \dot{\zeta}(t) = \lim_{\varepsilon \rightarrow 0} \begin{cases} 0 & t < 0 \\ \dot{\gamma} & 0 \leq t < \varepsilon \\ 0 & t \geq \varepsilon \end{cases}$$

$$\dot{\gamma}\varepsilon = \text{constant} = \gamma_0$$

Material Functions:

$G(t, \gamma_0) \equiv \frac{-\tau_{21}(t, \gamma_0)}{\gamma_0}$	First normal-stress relaxation modulus	$G_{\Psi_1} \equiv \frac{-(\tau_{11} - \tau_{22})}{\gamma_0^2}$
	Relaxation modulus	Second normal-stress relaxation modulus

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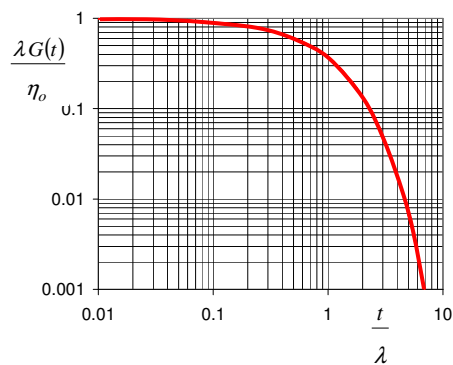
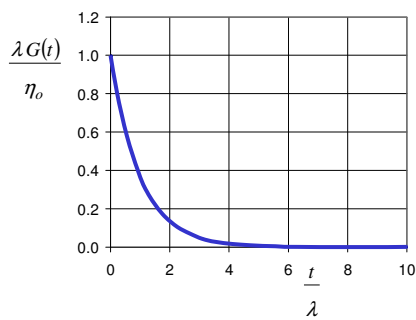
$$\underline{\underline{\tau}} + \frac{\eta_0}{G} \frac{\partial \underline{\underline{\tau}}}{\partial t} = -\eta_0 \underline{\underline{\gamma}} \quad \underline{\underline{\tau}}(t) = - \int_{-\infty}^t \left(\frac{\eta_0}{\lambda} \right) e^{-(t-t')/\lambda} \underline{\underline{\gamma}}(t') dt'$$

Shear start-up $\eta^+(t) = \eta_0(1 - e^{-t/\lambda})$ **Does** predict a gradual build-up of stresses on start-up.
 $\Psi_1^+(t) = \Psi_2^+(t) = 0$

Step shear strain $G(t) = \frac{\eta_0}{\lambda} e^{-t/\lambda}$ **Does** predict a reasonable relaxation function in step strain (but no normal stresses again).
 $G_{\Psi_1} = G_{\Psi_2} = 0$

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Step-Shear-Strain Material Function $G(t)$ for Maxwell Model



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Comparison to experimental data

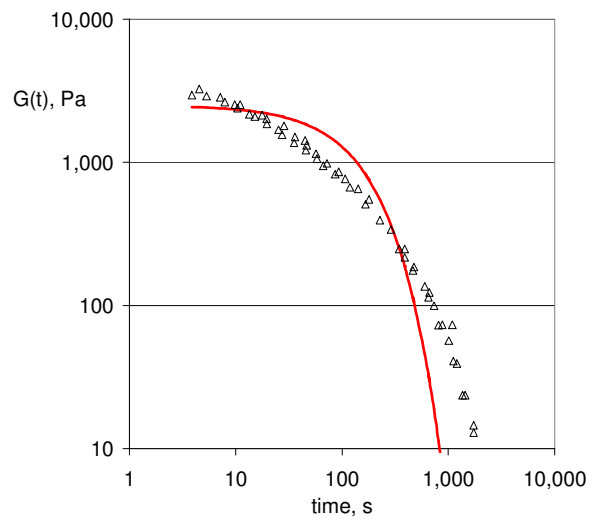


Figure 8.4, p. 274 data from Einaga et al., PS 20% soln in chlorinated diphenyl

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