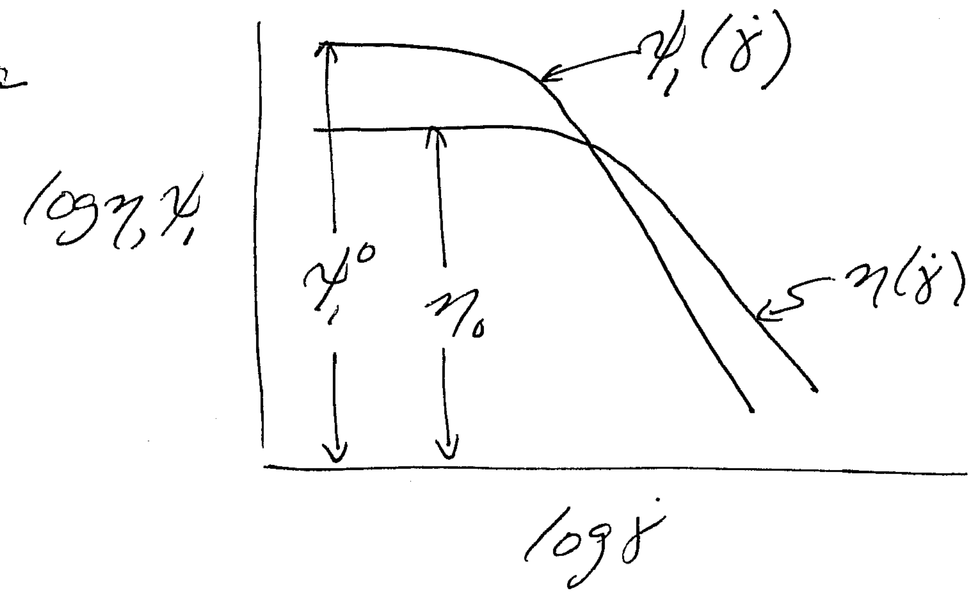
Exam 2
CM 4650
3 April 2007

1. Constitutive equations give stress + the stress tensor is independent of coordinate system. Therefore the constitutive equation must only be a function of things that are independent of coordinate system, like invariants.

2. a



2b

2c $\bar{\eta} = 3\mu$ Trouton viscosity

3. $$\underline{\underline{\tau}}(t) = -\eta\, \underline{\underline{\dot{\gamma}}}(t)$$

stress at current time

deformation gradient at current time

No, this cannot predict memory effects because the constitutive equation makes no reference to times other than the current time t. //

4. $$\underline{v} = \begin{pmatrix} -\dot{\varepsilon}_0 \frac{x_1}{2} \\ -\dot{\varepsilon}_0 \frac{x_2}{2} \\ \dot{\varepsilon}_0 x_3 \end{pmatrix}_{123}$$

$$\nabla \underline{v} = \begin{pmatrix} -\frac{\dot{\varepsilon}_0}{2} & 0 & 0 \\ 0 & -\frac{\dot{\varepsilon}_0}{2} & 0 \\ 0 & 0 & \dot{\varepsilon}_0 \end{pmatrix}_{123}$$

$$\dot{\underline{\underline{\gamma}}} = \nabla \underline{v} + (\nabla \underline{v})^T = \begin{pmatrix} -\dot{\varepsilon}_0 & 0 & 0 \\ 0 & -\dot{\varepsilon}_0 & 0 \\ 0 & 0 & 2\dot{\varepsilon}_0 \end{pmatrix}_{123}$$

$$|\dot{\underline{\underline{\gamma}}}| = \sqrt{\frac{6\,\dot{\varepsilon}_0^2}{2}} = \dot{\varepsilon}_0 \sqrt{3}$$

$$\underline{\underline{\tau}} = -m \dot{\gamma}^{n-1} \dot{\underline{\underline{\gamma}}}$$

$$\tau_{11} = \tau_{22} = -m\left(\dot{\varepsilon}_0\sqrt{3}\right)^{n-1}\left(-\dot{\varepsilon}_0\right)$$

$$= m\,\dot{\varepsilon}_0^{\,n}\left(\sqrt{3}\right)^{n-1}$$

$$\tau_{33} = -m\left(\dot{\varepsilon}_0\sqrt{3}\right)^{n-1} 2\dot{\varepsilon}_0$$

$$= -2m\,\dot{\varepsilon}_0^{\,n}\left(\sqrt{3}\right)^{n-1}$$

$$\underline{\underline{\tau}} = \begin{pmatrix} m\dot{\varepsilon}_0^{\,n}\,3^{\frac{n-1}{2}} & 0 & 0 \\ 0 & m\dot{\varepsilon}_0^{\,n}\,3^{\frac{n-1}{2}} & 0 \\ 0 & 0 & -2m\dot{\varepsilon}_0^{\,n}\,3^{\frac{n-1}{2}} \end{pmatrix}_{123}$$

$$\bar{\eta} = \frac{-(\tau_{33}-\tau_{11})}{\dot{\varepsilon}_0} = \frac{-\left(-2m\dot{\varepsilon}_0^{\,n}\,3^{\frac{n-1}{2}} - m\dot{\varepsilon}_0^{\,n}\,3^{\frac{n-1}{2}}\right)}{\dot{\varepsilon}_0}$$

$$\boxed{\bar{\eta} = 3m\dot{\varepsilon}_0^{\,n-1}\,3^{\frac{n-1}{2}} = m\dot{\varepsilon}_0^{\,n-1}\,3^{\frac{n+1}{2}}}$$

//

5.
$$\nabla \cdot \underline{v} = 0 \qquad \underline{v} = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}_{123}$$

$$\frac{\partial v_1}{\partial x_1} + \cancel{\frac{\partial v_2}{\partial x_2}} + \cancel{\frac{\partial v_3}{\partial x_3}} = 0$$

$$\boxed{\frac{\partial v_1}{\partial x_1} = 0}$$

$$\rho \left(\underbrace{\cancel{\frac{\partial \underline{v}}{\partial t}}}_{\text{steady}} + \underbrace{\cancel{\underline{v} \cdot \nabla \underline{v}}}_{\text{unidir}} \right) = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \underbrace{\cancel{\rho \underline{g}}}_{\text{neglect}}$$

$$\nabla p = \begin{pmatrix} \frac{\partial p}{\partial x_1} \\ \frac{\partial p}{\partial x_2} \\ \frac{\partial p}{\partial x_3} \end{pmatrix}_{123}$$

$$\nabla \cdot \underline{\underline{\tau}} = \frac{\partial}{\partial x_i} \underbrace{\hat{e}_i \cdot \tau_{pk} \hat{e}_p}_{\delta_{ip}} \hat{e}_k$$

$$= \frac{\partial \tau_{pk}}{\partial x_p} \hat{e}_k$$

$$\nabla \cdot \underline{\underline{\tau}} = \begin{pmatrix} \cancel{\frac{\partial \tau_{11}}{\partial x_1}} + \frac{\partial \tau_{21}}{\partial x_2} + \cancel{\frac{\partial \tau_{31}}{\partial x_3}} \\ \frac{\partial \tau_{12}}{\partial x_1} + \cancel{\frac{\partial \tau_{22}}{\partial x_2}} + \cancel{\frac{\partial \tau_{32}}{\partial x_3}} \\ \cancel{\frac{\partial \tau_{13}}{\partial x_1}} + \cancel{\frac{\partial \tau_{23}}{\partial x_2}} + \cancel{\frac{\partial \tau_{33}}{\partial x_3}} \end{pmatrix}_{123}$$

from $\underline{\underline{\dot{\gamma}}}$ we cancel these terms

$$\underline{\underline{\tau}} = -m\dot{\gamma}^{n-1}\,\underline{\underline{\dot{\gamma}}}$$

$0 = \nabla \cdot \underline{v}$

$$\nabla \underline{v} = \begin{pmatrix} \cancel{\frac{\partial v_1}{\partial x_1}} & \cancel{\frac{\partial v_2}{\partial x_1}} & \cancel{\frac{\partial v_3}{\partial x_1}} \\ \frac{\partial v_1}{\partial x_2} & \cancel{\frac{\partial v_2}{\partial x_2}} & \cancel{\frac{\partial v_3}{\partial x_2}} \\ \cancel{\frac{\partial v_1}{\partial x_3}} & \cancel{\frac{\partial v_2}{\partial x_3}} & \cancel{\frac{\partial v_3}{\partial x_3}} \end{pmatrix}$$

wide

$$\underline{\underline{\dot{\gamma}}} = \begin{pmatrix} 0 & \frac{\partial v_1}{\partial x_2} & 0 \\ \frac{\partial v_1}{\partial x_2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$|\underline{\underline{\dot{\gamma}}}| = \sqrt{\frac{\underline{\underline{\dot{\gamma}}} : \underline{\underline{\dot{\gamma}}}}{2}}$$

$$|\underline{\underline{\dot{\gamma}}}| = \pm \frac{\partial v_1}{\partial x_2}$$

$$\frac{\partial v_1}{\partial x_2} < 0 \quad \therefore \quad \boxed{\dot{\gamma} = -\frac{\partial v_1}{\partial x_2}}$$

$$\tau_{12} = \tau_{21} = -m\left(-\frac{\partial v_1}{\partial x_2}\right)^{n-1}\frac{\partial v_1}{\partial x_2}$$

$$= m\left(-\frac{\partial v_1}{\partial x_2}\right)^{n}$$

$$\nabla \cdot \underline{\underline{\tau}} = \begin{pmatrix} \frac{\partial}{\partial x_2}\left(m\left(-\frac{\partial v_1}{\partial x_2}\right)^{n}\right) \\ \cancel{\frac{\partial}{\partial x_1}\left(m\left(-\frac{\partial v_1}{\partial x_2}\right)^{n}\right)} \\ 0 \end{pmatrix}$$

not a function of x_1

Eqn of motion:

3-component: $\frac{\partial P}{\partial x_3} = 0$

2-component: $\frac{\partial P}{\partial x_2} = 0$

1- component:

$$\underbrace{\frac{\partial P}{\partial X_1}}_{\text{function of } X_1} = \underbrace{-\frac{\partial}{\partial X_2}\left(m\left(-\frac{\partial v_1}{\partial X_2}\right)^n\right)}_{\text{function of } X_2} = \lambda$$

$$\frac{dP}{dX_1} = \lambda$$

$$P = \lambda X_1 + C_1$$

$X = 0 \quad P = P_0$
$X = L \quad P = P_L$

$$\boxed{P = \left(\frac{P_L - P_0}{L}\right) X_1 + P_0}$$

$$\lambda = \frac{P_L - P_0}{L} < 0$$

$$\frac{d}{dX_2}\left(m\left(-\frac{dv_1}{dX_2}\right)^n\right) = -\lambda$$

$$m\left(-\frac{dv_1}{dX_2}\right)^n = -\lambda X_2 + C_2$$

BC: $X_2 = 0 \quad \frac{dV_1}{dX_2} = 0 \Rightarrow \boxed{C_2 = 0}$

$$\left(-\frac{dV_1}{dx_2}\right)^n = -\frac{\lambda x_2}{m}$$

$$-\frac{dV_1}{dx_2} = \left(-\frac{\lambda}{m}\right)^{\frac{1}{n}} x_2^{\frac{1}{n}}$$

$$V_1 = -\left(-\frac{\lambda}{m}\right)^{\frac{1}{n}} \frac{x_2^{\frac{1}{n}+1}}{\frac{1}{n}+1} + C_3$$

$$\boxed{V_1 = -\left(\frac{P_0 - P_L}{mL}\right)^{\frac{1}{n}} \left(\frac{1}{\frac{1}{n}+1}\right) x_2^{\frac{1}{n}+1} + C_3}$$

BC: $x_2 = H \quad V_1 = 0$ evaluate C_3

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Force on top wall:

$$F = \tau_{21}\Big|_{x_2=H} LW$$

$$\tau_{21} = m\left(-\frac{dv_1}{dx_2}\right)^n$$

$$-\frac{dv_1}{dx_2} = \left(\frac{P_0-P_L}{Lm}\right)^{\frac{1}{n}} x_2^{\frac{1}{n}}$$

$$= \cancel{m}\left(\frac{P_0-P_L}{L\cancel{m}}\right) H$$

$$F = \frac{\Delta P H \cancel{L} W}{\cancel{L}} = \boxed{\Delta P H W = F}$$